Influencing the Misinformed Misbehaver: An Analysis of Public Policy towards Uncertainty and External Effects[*]

Francesca Barigozzi[†] Bertrand Villeneuve[‡]

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Abstract

We study a situation in which government influences consumers' behaviors by providing both information and incentives. We develop the case of consumption choice in the presence of uncertainty and external effects. The instruments used by the government are information campaigns and taxes. A difficulty arises because the government would like to bolster these less than perfectly effective tools by delivering biased information to the misinformed misbehaver. We study the equilibrium trade-off between informing and offering incentives. Environmental tax policy, anti-smoking campaigns and policy against over-consumption of antibiotics serve as illustrations. Technically we solve a model of signaling cum cheap-talk.

Keywords: information campaigns, tax policy, cheap talk, signaling, skeleton.

JEL: I18, H30, D82.

1 Introduction

As for the "mad cow" disease and other hotly debated issues concerning public health, food safety and the environment, risk controversies have mushroomed.

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[†] University of Bologna.

[‡] CEA and IDEI, University of Toulouse. Corresponding author: Bertrand Villeneuve, IDEI, Université de Toulouse 1, 21 allée de Brienne, 31000 Toulouse, FRANCE. E-mail: bertrand.villeneuve@cict.fr.

Since policy makers must often assess and communicate such risks, the individuals' con…dence in government or other authorities is a decisive component of policy-making. Our work focuses on communication as disclosing the con‡icts between a benevolent authority and consumers. Two ingredients are indispensable: the public's ignorance of the risk to be regulated and the impossibility for individuals alone to rightly internalize certain negative consequences of their actions. In isolation, each is relatively easy to solve: the former needs information provided to the public; the latter, optimized incentives. But in combination these remedies interfere with one another and result in political confusion when incentives and coercive instruments are defective.

In the present paper, the policy-making process is analyzed as a game in which government wants to in‡uence consumers' behaviors through both tax policy and information campaigns, and where rational consumers react in a Bayesian manner. Instruments being imperfect, the government is often tempted to "improve" behavior by providing biased information. Con…dence, we show, is not easily controlled. Depending on the coordination between government and consumers, the same background data can produce various policies and real e¤ects. We determine the structure of practicable policies and discuss the trade-o¤ between vagueness in communication and distortion of incentives.

Our model deals with policies that a¤ect the consumption of commodities that are detrimental to consumers' welfare both individually and collectively. Typically, side e¤ects are due to individual consumption; external e¤ects are due to overall consumption in the economy. In the case of tobacco and alcohol, one can readily distinguish between disease related to individual consumption and the social damage from passive smoking or the cost to the health care system (not to mention psychosocial issues like drunk driving and addiction). Broad-spectrum antibiotics display (apart from the obvious bene…ts) this double negative impact. At the individual level they clear the way to opportunistic infection by more resistant germs;[1] and at the societal level, they enhance the resistance of the germs involved in contagious diseases.

These two types of negative e¤ects explain why, without government intervention in the form of information or incentives, consumers may not consume e¢ciently. First, side e¤ects are not necessarily recognized by consumers. For example, the risk smokers perceive can be under- or over-estimated (Viscusi 1990); likewise, the real magnitude of side e¤ects of antibiotics is no more than a vague notion for most people. Second, external e¤ects (e.g. passive smoke, or the

[1] Some broad-spectrum antibiotics decrease the individual's immunological response, and as a consequence new diseases can arise. For example, many antibiotics based on penicillin are used to treat diseases like bronchitis, otitis and tonsillitis caused by di¤erent bacteria (staphilococcus aureus, haemo…lus in‡uenzae, streptococcus pneumoniae). Possible side e¤ects of penicillin consumption are candida albicans and herpes. See Levy (1992) for the medical viewpoint and Brown and Layton (1996) for an excellent economic analysis of the external e¤ects.

rise of resistant strains) are very largely ignored by consumers in the absence of incentives such as taxes, norms, controls.

Political attitudes towards tobacco are typical of the schizophrenia we discuss in the present work. Efficient taxation is, generally, difficult to establish, but compared to others levies, tobacco taxes are an easy source of funds. Government might try to optimize both health and budgetary objectives by manipulating consumers' beliefs on the individual consequences of smoking. Obviously, rational consumers form their opinion with this danger in mind, so the success of such attempts is uncertain.

In our model the tax system is inefficient in two respects: first individual consumption is unobservable, which imposes a linear tax. Second, rising funds generates a social welfare loss. This latter imperfection is measured by "the cost of public funds" which, depending on its sign, can push towards over- or under-taxation.

We assume that the government is benevolent (it maximizes the utility of the representative consumer), and better informed: the government has the means to access to the best available information. Moreover, informing the public is a never-ending task. We know that discouraging teenagers from smoking require renovated strategies year after year. Though one may think that the "society" is already saturated with information on the relationship between tobacco and cancer, each new cohort of consumers still has to be educated.

Nevertheless, the government confronts the following dilemma: taxes are imperfect instruments, and it is tempting to make them work better by disseminating biased information. This engenders a sort of paternalism: the government wants consumers to consume efficiently, but, being unable to commit to neutral and truthful information transmission, it may send interested messages.

One crucial aspect of the model is the analysis of the tax. The tax has two consequences: first, by modifying the price it provides an incentive to internalize the external effects; second, it acts as a signal of the value of side effects. In our view, in all circumstances where there is some imperfection that impedes economic efficiency, taxes must be understood as a metaphor for the entire comprehensive policy package, also comprising contracts, bans, standards and norms. Indeed, apart from taxes, anti-smoking measures comprise a large span of instruments, like advertising bans, prohibition of sales to minors, smoke-free areas, etc. All require implementation costs.

Information campaigns are conceived for transmitting government information to consumers. It is useful to distinguish between "cheap talk" à la Crawford and Sobel (1982) and "hard information". Examples of the former are warning labels on cigarette packs or on the hazards of alcoholic beverage.[2] The latter, that we

[2]A characteristic of these campaigns is that they have no direct impact on government's or consumers' utility. Note that our interpretation remains valid if information campaigns are costly, as long as the cost is independent of the message the authority decides to send. Since the diffusion cost of "smoke is detrimental to one's health" is the same as that of "smoke is

could also name education, implies that the government collects and presents detailed scienti…c evidence, that it employs other relays (academics, teachers, social workers, newspapers, etc.) and that consumers accept to spend time to learn this information. Given the di¤erence in costs, it is rational for the society to rely on cheap talk communication after a certain degree of educational e¤ort has been done. To focus on the impact and credibility of cheap talk, we do not model hard information, assuming that an exogenous level has been provided.

Cheap talk supposes that the literal meaning of the information conveyed is vague enough not to be falsi…able. Take the warning label "Seriously Harmful to Health" on cigarette packs. This is not false, but the exact nuance it carries is a matter of social convention (speci…cally, how cheap talk is interpreted). Like taxes, information campaigns are imperfect instruments of policy.[3]

We show, given benevolent authorities and rational consumers, that the major cause of trouble is lack of government credibility. By de…nition, if government could commit ex ante to inform truthfully, it would not encounter the di¢culties we discuss. Yet a distrustful attitude on the part of the public towards informed authorities is frequent: people often feel that the government's actions are motivated by economic interest more that by the public interest. Until recently, for example, cigarettes were produced by public monopolies in France and Italy, which has been severely criticized as an unsolvable con‡ict of interest. The di¤usion of information about the HIV-contaminated blood in the eighties in France and Germany, or "mad cow" disease in Europe, are also examples in which the ambivalent attitude of the governments has been repeatedly underlined. Separating the two functions (information and taxation) would be an attractive solution, if it really can improve the commitment of the public authorities. However, understanding the cause of the absence of commitment and proposing remedies is beyond the scope of this paper.

To solve the social game sketched here, we take an approach based on Bayesian equilibrium: people are not systematically fooled and the government tries to make the best of the instruments available. Rather than searching for the equilibria of a given economy, we solve the inverse problem: we de…ne the minimal amount of information (a skeleton) to describe an equilibrium and determine the full set of economies that admit a given skeleton as equilibrium. The skeleton approach helps understanding of the interaction between costly and free signals and opens the way to interesting comparative statics. It becomes relatively easy to see that cheap talk is almost useless in our model because costly signals (taxes)

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very detrimental to one's health", we can, without generality loss, normalize this cost to zero, if no-message is not an e¢cient choice.

[3]Crawford and Sobel (1982) show that the precision of the information conveyed by cheap-talk depends on the intensity of the con‡ict between the two parties' objectives. Quite recently, these ideas have been applied to political games in which a lobby tries to in‡uence policy makers. See Helpman (2000), Presidential Address at the Econometric Society World Congress, Seattle and Grossman and Helpman (2001), for surveys.

are available (see also Manelli 1996). We prove the uniqueness of the fully revealing equilibrium and characterize it in detail in a non-monotonic context that makes the method of Mailath (1987) inapplicable. We show in our context how more informative equilibria are more distorted.

The literature on multiple informative instruments is scarce. In a classical contribution to the rational foundations of advertising, Milgrom and Roberts (1986) model a firm that signals its product quality through price and dissipative advertising (burned money) to enhance consumers' willingness to pay for the product. Starting from the model of Crawford and Sobel, Austen-Smith and Banks (2000) show how burning money can improve cheap talk. They clearly show why the information transmitted can be perfect, and why the most informative equilibrium need not be the most efficient. In our model the government transmits information using both cheap talk and the tax (a costly signal). Though several of our arguments are not new, they require different proofs and interpretations. Indeed, tax distortions are not monotonic with the level of the tax, therefore signaling costs have a quite complex structure.

Plan Section 2 presents the terms of the policy dilemma in the case of commodities affecting health and the environment. Section 3 defines the equilibrium. The main body of the paper develops the methodology. After a few results on the structure of the government's preferences (Section 4), the analysis is developed in three steps. First we show that an equilibrium can be summed up by its "skeleton", i.e. a relatively small set of policies satisfying incentive compatibility for the sender (Section 5): Second, we show under what circumstances a given "skeleton" can be implemented in an equilibrium. This is crucial to getting insights into the structure of partially revealing equilibria (Section 6). Finally we characterize the game's unique fully-revealing equilibrium (Section 7).

2 The Model

2.1 The Consumers

Consumers live two periods and their period-2 utility is negatively affected by period-1 consumption x_1 (e.g. cigarettes). Preferences can be written as:

$$U[x_1] + x_2 - \theta x_1 - \eta \overline{x}_1 \qquad (1)$$

where U is the logarithmic utility function.[4] The consequences of x_1 on period-2 utility pass through two distinct channels:

[4] Most of the propositions in Section 4 (characterization of the equilibria) do not rely on this restriction on utility, as can be seen in the proofs, but it does improve the legibility of the explicit calculations of the first- and second-best.

- The term $-\mu x_1$ measures side effects (poor health) due to the consumer's own consumption in period 1. The intensity μ is not precisely known to consumers. The cumulative distribution function $F(\mu)$ and its density $f(\mu)$, both supported in $[\underline{\mu}, \overline{\mu}]$, represent consumers' priors on μ. In general, f is continuous and non-negative on the support.

- The term $-\eta \overline{x}_1$ indicates the negative externality (passive smoking and added cost to the health care system) that depends on $\overline{x}_1$, i.e. average period-1 consumption in the economy. The intensity η is supposed to be known to all the agents.[5] The consumer does not internalize the social consequences of x_1. This is because there are a large number of atomistic consumers in the economy: each knows that he would affect the externality only marginally.

Let t be the linear tax rate set by the government. The representative consumer, not internalizing the externality η, solves:

$$(2) \qquad \begin{cases} \max\limits_{x_1, x_2} E\left[U[x_1] + x_2 - \mu x_1\right] \\ \text{s.t.} : (p_1 + t)x_1 + p_2 x_2 = W \end{cases}$$

where the expected value of utility is conditional on the consumer's information; p_1 and p_2 are the prices for, respectively, period-1 and -2 consumption, and W is the consumer's endowment.

To further simplify the program we normalize p_2 to 1 and p_1 to 0 without loss of generality since the support of μ can be translated to account for the price, which is exogenous. Then we substitute the budget constraint into the objective function and drop the subscripts to write the first period consumption as x. The simplified consumer's program is:[6]

$$(3) \qquad \max_{x} E\left[U[x] - (\mu + t)x\right]$$

As a consequence, consumption choice x^* depends on the consumer's information and on the tax rate t:

$$(4) \qquad x^*[E\mu, t] \text{ solves } U'[x] = E\mu + t$$

that is

$$(5) \qquad x^*[E\mu, t] = \frac{1}{E\mu + t}$$

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[5] An alternative model could put uncertainty on η. In general, though, this uncertainty alone would not exhibit the sort of conflict we are pointing at, since the consumer's behavior is not affected by the intensity of the externality. In our specification, consumption does not even depend on η.

[6] Notice that the linear part in the preferences also represent the utility from goods other than x.

2.2 Social Welfare and the Marginal Cost of Public Funds

The government maximizes a social welfare function that corresponds to the consumers' utility once the externality and the exact value of side effects are taken into account. Calculating how the commodity x should be taxed, the government is constrained by the fact that the tax t is linear because individual consumption is unobservable. Moreover, rising funds generates a welfare cost since taxes are imperfect instruments. In addition the government is hampered by its inability to commit to a policy that informs truthfully on the value of side effects θ. In other words, the government is benevolent in that it evaluates consumption in consumers' best interest, but opportunistic because it does not value truthful information per se, and would deceive consumers provided this induces "better" behavior (and reduces the distortions caused by the tax): in short it practices a variety of paternalism.

All consumers being identical, in equilibrium $\bar{x} = x$, and the government's objective function can be represented as:

$$U[x] - (\theta + \delta)x + S - (1 + \lambda)R \tag{6}$$

where S is the consumer's surplus from public expenditures R: The government raises the exogenous amount R with general taxation (income-, capital-tax, or other levies) at the welfare cost $(1 + \lambda)R$; with $\lambda > -1$: Most partial equilibrium models call parameter λ the "shadow cost" of public funds; it represents the distortion due to the raising of fiscal revenue.[7]

In our model, R and S remain constant, while a new tax on the good x is added to existing taxes. The revenue tx being small compared to R; our assumption that λ remains constant is reasonable. As modern public finance theory has shown, no general conclusion can be drawn about the sign of the shadow cost of taxation when a revenue-neutral substitution between different taxes is implemented. The sign of λ is not restricted *a priori* and depends on the structure of preexisting taxes (in particular their efficiency) and on how they interact with the new tax.[8]

When the government introduces a tax t on good x and revenue tx is devoted to reducing preexisting taxes, (6) becomes, after simplification:

$$U[x] - (\theta + \delta - \lambda t)x + S - (1 + \lambda)R \tag{7}$$

Comparing (3) and (7), we see that the government's program differs from the consumer's in three ways: superior information on θ; internalization of δ; and

[7] See, for example, the shadow cost of public funds used in the theory of regulation (Laffont and Tirole 1993). In general equilibrium models of taxation (e.g. Ramsey), λ would be the (endogenous) Lagrange multiplier associated with the government's budget constraint. Under some regularity conditions, the Lagrange multiplier is equivalent to what the theory of cost-benefit analysis calls the shadow cost of a marginal change in a public project. See Drèze and Stern (1987).

[8] For a concise discussion, see Ballard and Fullerton (1992) and Goulder (1995).

the presence of λ in the government's objective function. The latter difference reflects that the consumer does not internalize the fact that his contribution tx will replace revenue from other potentially distortionary taxes in the economy.

The case $\lambda > 0$ (preexisting taxes inflict a welfare cost larger than R) relates to the recently debated "double dividend" effect (Pearce 1991). According to this literature, a revenue-neutral substitution of environmental taxes for other taxes might offer a double dividend: not only does it (i) improve the environment but it also (ii) reduces the costs of the tax system through cuts in distortionary taxes (see Goulder 1995). To intuit this result, assume that x and the other taxed goods (labor included) are gross substitutes. In that case, typically, the tax t reduces the consumption of x and increases the consumption of the other taxed goods. Thus total fiscal revenue increases and taxes on the other goods can be reduced, which attenuates distortions.[9;10] Notice that we could also reason in terms of relative efficiency: when $\lambda > 0$ a tax on good x is relatively less distortionary than preexisting taxes; when $\lambda < 0$; it is more distortionary. As far as tobacco is concerned, $\lambda > 0$ is the most plausible.

Dropping constant terms, we get a reduced form of the government's objective function:

$$SW[x;t;\theta] \equiv U[x] - (\theta + \delta - \lambda t)x \tag{8}$$

2.3 Constrained Efficient Allocations

The first-best allocation is defined as the allocation that maximizes consumers' utility when θ is known, δ internalized, and the economy is not distorted ($\lambda = 0$): This gives $x_{FB}(\theta) = \frac{1}{\theta+\delta}$ (see equation (3) with δ instead of t). This allocation can be decentralized even for unobservable x with the Pigovian linear tax $t = \delta$:

The second-best allocation is defined as the best the government can attain if consumers are perfectly informed on θ when (i) it is constrained to a linear tax on x; and (ii) the marginal cost of public funds is not zero. This can be written as follows:

$$\begin{cases} \max\limits_{t} U[x] - (\delta + \theta - \lambda t)x \\ \text{s.t.} : x = \frac{1}{\theta+t} \end{cases} \tag{9}$$

where the constraint on x corresponds to consumers' reaction function (5) when θ is known. Hence the second-best consumption and tax rate are:

$$\begin{cases} x_{SB}(\theta) = \frac{1}{\delta+(1+\lambda)\theta} \\ t_{SB}(\theta) = \delta + \lambda\theta \end{cases} \tag{10}$$

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[9] Complementarity between x and the other taxed goods allows the same reasoning to hold when $\lambda < 0$.

[10] The same reasoning in terms of substitutability and complementarity between x; the other goods, and the public project (financed by R) applies. In other words, the cost of public funds also depends on the interaction between the public expenditures and the taxed activities.

Because of the "double dividend", the second-best tax is higher than the first-best when $\lambda > 0$. The opposite holds for $\lambda < 0$. For a given λ, the tax rate is strictly increasing (decreasing) with respect to θ when $\lambda > 0$ ($\lambda < 0$). This implies that the tax rate is potentially informative on the value of side effects.

Straightforward calculations lead to a sort of Ramsey-Boiteux pricing rule:[11]

$$\frac{t_{SB}(\theta) - \bar{t}}{t_{SB}(\theta)} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon} \tag{11}$$

where $\varepsilon \equiv -\frac{\partial x^*}{\partial t}\frac{t}{x^*} = \frac{t}{\theta+t}$ is the tax elasticity of demand, and $\bar{t} = \frac{\eta}{1+\lambda}$. As tax elasticity is decreasing in θ for all t, (11) shows that for positive λ, the stronger the side effects, the higher the tax. Larger side effects discouraging consumption more, fiscal revenue is partly preserved by larger taxes.[12] For negative λ, the opposite holds.

3 The Influence Game

The timing of the model is as follows: first the government observes θ; then it chooses its policy, and finally the consumer, observing the policy, updates his beliefs on θ and chooses his consumption level.

A policy $P = (m, t) \in M \times \mathbb{R}$ is composed of the tax rate t and a (cheap talk) "message" m selected from a certain large set, M. Through the choice of a policy P, the government wants to induce the consumer to approach efficient consumption. The tax has the two-fold role of incentive and information; cheap talk can only transmit information. We can think of m as composed of a "sentence" like labels on cigarette packs, or more generally warning adverts against smoking and alcoholic beverages. We assume that M is broad enough to say what needs to be said; it might be composed, say, of all reasonably short utterances (see, e.g., Farrell and Rabin 1996 on what cheap talk is and is not). It is useful, at this point, to make a distinction between the message the government sends and the consumers' interpretation of it at equilibrium. What really matters is not the message itself but the way the consumer understands the policy. To be clearer, whatever the phrasing of the communication, we concentrate on the meaning (the revised $E\theta$) that the consumer assigns to every policy.[13]

[11] A similar expression can be found in Sandmo (1975). See also Bovenberg and van der Ploeg (1994).

[12] A consequence is that the higher the risk associated with smoking, the higher the tax.

[13] As an example, let m_1 and m_2 denote two messages sent in a fully-revealing equilibrium. Assume that m_1 corresponds to the word "harmful" and m_2 corresponds to the word "hazardous". This is an equilibrium as long as the receiver understands this language and assigns to the message "harmful" the meaning, say "$\theta = \theta_1$", and to the message "hazardous" the meaning, say "$\theta = \theta_2$", where θ_1 and $\theta_2 \in [\underline{\theta}, \bar{\theta}]$.

After observing the policy, the consumer updates his priors which are then denoted by $\mu(P)$ (with $\mu(P) \in \Delta([\underline{\theta};\bar{\theta}])$; the set of probability distributions over $[\underline{\theta};\bar{\theta}]$). We denote $E(\theta|P)$ by $\hat{\theta}(P)$.

Definition 1 A perfect Bayesian equilibrium (PBE) of the game is a pure strategy P mapping $[\underline{\theta};\bar{\theta}]$ into $M \times \mathbb{R}_+$ and a belief μ mapping $M \times \mathbb{R}$ into $\Delta([\underline{\theta};\bar{\theta}])$ such that:

1. Policies are optimal given beliefs: for each $\theta \in [\underline{\theta};\bar{\theta}]$; $P(\theta)$ solves

$$\max_P SW[x^*[\hat{\theta}(P);t];t;\theta]. \tag{12}$$

2. Beliefs are rational given equilibrium policy: for each P, $x^*[\hat{\theta}(P);t]$ solves

$$\max_x \int_{\underline{\theta}}^{\bar{\theta}} [U[x] - (\theta + t)x]\mu(\theta|P)d\theta; \tag{13}$$

where $\mu(\theta|P) \equiv \frac{\mathbb{I}_{\{P(\theta)=P\}}\Delta f(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \mathbb{I}_{\{P(s)=P\}}\Delta f(s)ds}$; $\mathbb{I}$ being the indicator function.

Out of equilibrium beliefs are not restricted.[14]

4 Government Policy Preferences

In any equilibrium, the larger the side effects, the lower the social welfare: the consumer's rationality prevents the government from transforming lead into gold by clever communication strategies. To see that, let θ_1 and θ_2 be two possible states of the world, $P_1 = (m_1; t_1)$ and $P_2 = (m_2; t_2)$ two equilibrium policies, and x_1 and x_2 the consumption levels induced. If $\theta_1 < \theta_2$; then $U[x_2] - (\eta + \theta_1 - \lambda t_2)x_2 > U[x_2] - (\eta + \theta_2 - \lambda t_2)x_2$. On the other hand, the incentive constraint of the type-θ_1 social planner reads: $U[x_1] - (\eta + \theta_1 - \lambda t_1)x_1 > U[x_2] - (\eta + \theta_1 - \lambda t_2)x_2$. We get: $U[x_1] - (\eta + \theta_1 - \lambda t_1)x_1 > U[x_2] - (\eta + \theta_2 - \lambda t_2)x_2$. Thus the social planner's pay-off decreases with respect to the side effects. In other words, we preclude any perverse mechanism whereby propaganda can make less desirable states of the world (larger side effects) preferable.

We analyze now the government's incentive to distort taxes and beliefs, i.e. the reasons why consumers are likely to be suspicious of the government's actions and claims, and how suspicious.

Remark 1 In equilibrium, any policy P can be analyzed without loss of insight as a pair $(\hat{\theta}; t)$, where t is the tax rate, and $\hat{\theta}$ the belief associated to the policy.

[14] Proposition 2 implies that integrals are always well-defined.

We define $\overline{SW}[t;\hat{\theta};\theta] \equiv SW[x^*[\hat{\theta};t];t;\theta]$ as the value of a policy characterized by the belief-tax pair $(\hat{\theta};t)$ for a government of type θ: Reasoning directly on belief-tax pairs allows a simpler analysis of incentive constraints, independently of the cheap talk message chosen. Indeed, incentive compatibility for $P(\theta) = (\hat{\theta};t)$ and $P(\theta') = (\hat{\theta}';t')$ can clearly be checked by comparing $\overline{SW}[t;\hat{\theta};\theta]$ with $\overline{SW}[t';\hat{\theta}';\theta]$; and $\overline{SW}[t;\hat{\theta};\theta']$ with $\overline{SW}[t';\hat{\theta}';\theta']$:

Policies are restricted to induce finite consumption, thus feasible policies are such that $t+\hat{\theta} > 0$. Indifference curves are not monotonic with respect to the tax or the belief. The following proposition gives us the properties required to proceed with the analysis of incentive compatibility.

Proposition 1 1. For all θ; the upper contours of $\overline{SW}$ with respect to t and $\hat{\theta}$ are convex.

2. (Single-crossing) For all $(\hat{\theta};t)$; the tangent of the indifference curve for type θ passing through $(\hat{\theta};t)$ turns continuously clockwise if $\lambda > 0$ (anti-clockwise if $\lambda < 0$) as θ increases. Moreover, indifference curves related to two different types cross once at most.

Proof. 1. It suffices to verify that the utility is quasi-concave. To do this, we check that the successive principal minors of the bordered Hessian matrix have alternate signs (odd principal minors must be positive). The bordered Hessian matrix is:

$$\begin{bmatrix} 0 & \frac{\eta+\theta-t-(1-\lambda)\hat{\theta}}{(t+\hat{\theta})^2} & \frac{\eta+\theta-(1+\lambda)t-\hat{\theta}}{(t+\hat{\theta})^2} \\ \frac{\eta+\theta-t-(1-\lambda)\hat{\theta}}{(t+\hat{\theta})^2} & -\frac{2\eta+2\theta-t-(1-2\lambda)\hat{\theta}}{(t+\hat{\theta})^3} & -\frac{2\eta+2\theta-(1+\lambda)t-(1-\lambda)\hat{\theta}}{(t+\hat{\theta})^3} \\ \frac{\eta+\theta-(1+\lambda)t-\hat{\theta}}{(t+\hat{\theta})^2} & -\frac{2\eta+2\theta-(1+\lambda)t-(1-\lambda)\hat{\theta}}{(t+\hat{\theta})^3} & -\frac{2\eta+2\theta-(1+2\lambda)t-\hat{\theta}}{(t+\hat{\theta})^3} \end{bmatrix} \tag{14}$$

The first principal minor is equal to zero, the second is negative, and we find $\frac{\lambda^2}{(t+\hat{\theta})^4}$ for the third, which gives result required.

2. The problem is that tangents are sometimes vertical (MRS pass from positive to negative value or the other way round). Nevertheless, we see from the first line of matrix (14)

$$\left.\frac{dt}{d\hat{\theta}}\right|_{\overline{SW}=\text{constant}} = -\frac{\eta+\theta-(1+\lambda)t-\hat{\theta}}{\eta+\theta-t-(1-\lambda)\hat{\theta}} \tag{15}$$

Its derivative with respect to θ is

$$-\frac{\lambda(t+\hat{\theta})}{(\eta+\theta-t-(1-\lambda)\hat{\theta})^2} \tag{16}$$

which is negative (positive) for $\lambda > 0$ ($\lambda < 0$) for $t+\hat{\theta} > 0$. Tangents to indifference curves being vertical whenever $\delta + \theta - t - (1 - \lambda)\hat{\theta} = 0$, the claim is correct.

If one sets aside domain restrictions (here $t + \hat{\theta} > 0$), upper contours are closed, meaning that two indifference curves related to two different types, if ever they cross, cross twice at least. But in the relevant range, two curves always cross in the same way: the standard single-crossing argument works. ■

The unconstrained optimal policy of the government is $(\hat{\theta} = -\frac{\delta+\theta}{\lambda}, t = \frac{\delta+\theta}{\lambda})$, the singular point where both the numerator and the denominator of (15) are equal to zero. The associated belief is in general out of the support, which is clearly not consistent with the Bayesian behavior of the consumers. The optimum with $t = 0$ is $\hat{\theta} = \theta + \delta$: if taxation is impossible, it would be optimal to set beliefs in such a way that consumers spontaneously internalize the externality. This is not Bayesian either. In the same vein, let us remark that the pervasive opinion that consuming a lot of antibiotics may cause individual resistance to the treatment is unfounded. (Actually the problem is the resistance acquired by the germs, which concerns the society rather than the individual strictly speaking). Authorities (which may not be directly responsible for this belief) are clearly tempted not to bother correcting it, since it serves (at a low cost) the practical goal of curbing consumption.

For all θ, tangents to indifference curves are horizontal along the straight line $(1+\lambda)t+\hat{\theta} = \delta+\theta$, and vertical along the straight line $t+(1-\lambda)\hat{\theta} = \delta+\theta$. Figure 1 shows the government's indifference curves for $\delta = 1$, $\underline{\theta} = 0$, $\bar{\theta} = 1$, $\lambda = .7$ and $\theta = 1$.

Insert Figure 1 here.

We prove that the second-best policy is not implementable in a Bayesian equilibrium. Suppose the consumer thinks that the government is playing the second-best strategy. The tax schedule t_{SB} being invertible, if $t_{SB}(\theta)$ is imposed, the individual can infer θ unambiguously. Proposition 1 implies that the government faces strong incentives to provide biased information.

Corollary 1 The second-best allocation is never an equilibrium if $\lambda \neq 0$.

In fact, at $(\theta, t_{SB}(\theta)) = (\theta, \delta + \lambda\theta)$, for all θ, the tangent of the indifference curve of the government is vertical (see point 2 in the Proposition): small changes in the tax have only second-order effects, whereas small changes in the beliefs have first-order effects on the government's objective. Consequently, if $\lambda > 0$, any policy close to the second-best $(\theta, \delta + \lambda\theta)$ but with $\hat{\theta} < \theta$ is preferred; this holds for a second-best policy associated with a slightly lower type. As far as commodities like tobacco are concerned, this means that lowering slightly the tax has negligible negative direct impacts on tax revenue, while the positive consequence on beliefs is sizable (consumers become more optimistic and then consumption increases). If $¸ < 0$; second-best policies associated with slightly higher types are preferred. In any case, the second-best allocation is not incentive-compatible, which con…rms that the government has a strong incentive to bias its information campaign.

Note in contrast that when $¸ = 0$; the …rst-best allocation is implementable in a PBE. Indeed, given that the government has no incentive to lie (compare (3) and (8) for $¸ = 0$), the tax is speci…cally used to internalize the externality ($t = ´$), but since the tax rate is uninformative on $µ$, cheap talk must be used to eliminate asymmetric information. With a slight abuse, equilibrium policies can be written as $P = (\hat{µ} = µ; t = ´)$; where information is fully transmitted. This is, of course, a very particular case.

Now we are able to summarize the trade-o¤ between implementing e¢cient taxes and transmitting truthful information. If the government could ex ante commit to truthful information transmission ($\hat{µ} = µ$), the second-best tax could be implemented. The resulting policy would be the second-best policy $P_{SB} = (\hat{µ} = µ; t = t_{SB})$: If the government could ex ante commit to the (well-chosen) tax $\bar{t} = \frac{´}{1+¸}$;[15] the information could be truthfully transmitted through the cheap talk message. The resulting policy would be the "committed honesty" policy $P_{CH} = (\hat{µ} = µ; t = \bar{t})$: In general, none of these policies is implementable in the absence of commitment. With the second-best policy, the tax is e¢cient, but incentives to provide biased information are systematic. With the committed honesty policy, truthful information is optimal, but incentives to change taxes are systematic. See Figure 1. We retrieve this con‡ict in the analysis of fully revealing equilibria (section 7). Notice that, absent imperfections ($¸ = 0$); these policies become identical, equivalent to …rst-best and implementable.

5 Skeletons

Description of all the equilibria given the prior type distribution is di¢cult. So rather than look for equilibria in the traditional way for signalling games, we introduce a di¤erent technique. That is we solve the inverse problem: we …nd the set of types and the distributions of types that are consistent with a certain equilibrium allocation. This approach has some relationship with mathematical tools mostly used in imagery (namely Voronoi diagrams, and their dual, Delaunay triangulation) from which we borrow our terminology (the skeleton).[16] The analogy is the following: given a partition of the types, types in each subset ap-

[15] $\bar{t}$ also serves as the reference cost in the Ramsey-Boiteux equation (11).

[16] The idea is basically the following: the Voronoi diagram of a point set P is a subdivision of the plane with the property that the Voronoi cell of point p contains all locations that are closer to p than to every other point of P. The points of P are also called Voronoi generators. Each edge of a Voronoi cell is the bisector of the connection of p to the corresponding neighbour cell. See http://www.voronoi.com/ for theory, algorithms, and examples of applications.

plying the same (unknown) policy, and two different subsets applying different policies, one may want to inquire into the underlying policies. Conversely, given a certain set of policies, and given that the government responds to its incentives, one may be interested in the types that have to be associated with each policy. In all these problems, preferences can be seen as a measure of distance.

The well-known result of Crawford and Sobel (1982, henceforth CS) that equilibria of cheap talk games are "partition equilibria" is generalized to our multiple instruments setting in the following proposition. See also Austen-Smith and Banks (2000).

Proposition 2 *Any equilibrium allocation can be implemented in an equilibrium in which there exists a partition of $[\underline{\theta}; \bar{\theta}]$ into a set of intervals $\{I_k\}_{i \in K}$ (K is a minimal set of indices) and a set of policies $\{P_k\}_{k \in K}$ such that (i) the policy chosen in I_k is P_k; and (ii) $k \neq k'$ implies $I_k \neq I_{k'}$ and $P_k \neq P_{k'}$. Moreover, the effects of policy P_k are entirely characterized by the pair $(\hat{\theta}_k; t_k)$, where $\hat{\theta}_k \equiv E(\theta|P_k) = E(\theta|I_k)); \forall k$.*

Proof. In this proof, optimal is used in the weak sense. In any PBE, for all P that are equilibrium actions, the set of types for which P is optimal is a convex subset of $[\underline{\theta}; \bar{\theta}]$. To see this, consider the sender's incentive constraint in a given equilibrium. Type θ will prefer policy $P_1 = (t_1; m_1)$ to any $P_2 = (t_2; m_2)$; implying, respectively, consumptions x_1 and x_2, if and only if:

$$\text{(17)} \quad \begin{aligned} U[x_1] - (\eta + \theta - \lambda t_1) x_1 &\geq U[x_2] - (\eta + \theta - \lambda t_2) x_2 \;, \\ \theta(x_2 - x_1) &\geq U[x_2] - U[x_1] + \eta(x_1 - x_2) + \lambda t_2 x_2 - \lambda t_1 x_1 \end{aligned}$$

This equation defines either a half straight-line in the space of types ($x_1 - x_2 \neq 0$) or the whole real line ($x_1 = x_2$). From this, it follows that if policy P is optimal for two values of θ; then it is optimal for any type that lies between these two values.

Let us denote by $(\theta_1; \theta_2)$; with $\theta_1 \neq \theta_2$; an interval in which P_1 is optimal. We now check that there is only one optimal policy in the interval. Suppose, for purpose of argument, that this is not the case, e.g. $\exists \theta \in (\theta_1; \theta_2)$ for which both P_1 and $P_2 (\neq P_1)$ are optimal. Equation (17) thus becomes

$$\text{(18)} \qquad \theta(x_2 - x_1) = U[x_2] - U[x_1] + \eta(x_1 - x_2) + \lambda t_2 x_2 - \lambda t_1 x_1$$

Now either $x_1 \neq x_2$, and, according to (17), P_1 is strictly preferred to P_2 on one side of θ; and P_2 is strictly preferred to P_1 on the other side, which is in contradiction with our assumption that P_1 is optimal on $(\theta_1; \theta_2)$; or else $x_1 = x_2$; which implies in turn that $t_1 = t_2$; and (given that consumptions are only a function of the tax and the beliefs) that P_1 and P_2 imply the same beliefs. In this case, P_1 and P_2 are the same in terms of tax and beliefs. Though they may differ in their cheap talk dimension, they can be seen as identical, and Remark 1 shows why.

If an equilibrium allocation were not implementable by a strategy based on a partition into intervals, then the latter result would be false. Hence, our claim is proven. ■

Proposition 2 suggests that only a "few" policies, compared to the "number" of types, can be considered: the updated beliefs and the tax rates associated with one of the intervals of the partition. Proposition 3 below is a reciprocal for which we need the following definition.

Definition 2 (Skeleton) Let $\{\theta_k\}_{k\in K}$ be a closed subset of $[\underline{\theta};\overline{\theta}]$ in which $k \neq k'$ implies $\theta_k \neq \theta_{k'}$ (K is a minimal set of indices); and let $\{t_k\}_{k\in K}$ be a set of real numbers. $\{(\theta_k; t_k)\}_{k\in K}$ is said to be a skeleton if and only if $\forall k; k' \in K; \overline{SW}[t_k;\theta_k;\theta_k] \geq \overline{SW}[t_{k'};\theta_{k'};\theta_k]$ (incentive compatibility).

Only "few" incentive compatibility constraints need to be checked to ensure that an allocation is an equilibrium, since we ignore types that are not in the skeleton.

Proposition 3 Let $\mathcal{F}$ be the set of type distribution F such that the skeleton $\{(\theta_k; t_k)\}_{k\in K}$ is an equilibrium set of policies. There exists a partition of $[\underline{\theta};\overline{\theta}]$ into a set of intervals $\{I_k\}_{k\in K}$ with $\theta_k \in I_k$ such that: $\forall F \in \mathcal{F}; \forall k;\ t(\cdot) = t_k$ over I_k and $E(\theta | I_k) = \theta_k$.

Proof. By convention, we denote the lowest element of $\{\theta_k\}_{k\in K}$ as θ_1; and the highest as θ_∞. Given θ_k, we define its successor in $\{\theta_k\}_{k\in K}$ as $\theta_{k+1} \equiv \min_{k'\in K}\{\theta_{k'} \geq \theta_k\}$ (this "+1" is just a convention, inspired by the fact that when K is finite, it can be reformulated as a set of successive integers). The type θ_{k+1} is well defined since a skeleton is closed.[17] We reason on incentive compatibility.

If $\theta_{k+1} \neq \theta_k$, we denote by τ_k a type which is indifferent between $P(\theta_k)$ and $P(\theta_{k+1})$; i.e. $\overline{SW}[t_k;\theta_k;\tau_k] = \overline{SW}[t_{k+1};\theta_{k+1};\tau_k]$. Given the single crossing property, and given the continuity of the government's welfare function with respect to the true type, τ_k is unique and belongs to $[\theta_k;\theta_{k+1}]$. We define $I_k = (\tau_{k-1};\tau_k]$. If the successor of θ_k is θ_k itself (this happens if θ_k is, on the right, an accumulation point in $\{\theta_k\}_{k\in K}$), then $I_k = \{\theta_k\}$. The lower bound of the lowest interval (i.e. containing θ_1) is $\underline{\theta}$, and the upper bound of the upper interval (containing θ_∞) is $\overline{\theta}$. Given Proposition 2, $t(\cdot) = t_k$ over I_k for all k is incentive-compatible. Finally, to ensure that the equilibrium beliefs of the consumer are Bayesian, it is necessary and sufficient that $F(\cdot)$ be such that $E(\theta | I_k) = \theta_k$. ■

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[17] Notice that we assume that $\{\theta_k\}_{k\in K}$ is closed only to simplify our reasoning. This assumption is in fact without loss of generality: if an accumulation point of $\{\theta_k\}_{k\in K}$ were missing (i.e. if $\{\theta_k\}_{k\in K}$ were not complete), we could add it to $\{\theta_k\}_{k\in K}$, with a corresponding accumulation point in $\{t_k\}_{k\in K}$. Due to the continuity of the incentive constraints, incentives are not reversed, and the skeleton is completed.

The type support can be divided into intervals in which the strategy is pooling, and Proposition 3 specifies the restrictions on the "flesh" (the distribution F) that may be put on the "bones" (the skeleton) to obtain an equilibrium. Conditional expectations (with respect to the policy, or to the interval) are independent of one another. The probability associated with the interval I_k not being constrained, $F \in \mathcal{F}$ can be chosen to be as smooth as wanted.

Figure 2 summarizes the notation and the main properties of the skeleton.

Insert Figure 2 here.

Corollary 2 If two different intervals are associated with two different tax rates, then the tax rate is sufficiently informative for the consumer, and the message can be ignored. If there exists $k \neq k'$ such that $t_k = t_{k'}$; then messages are indispensable to signal the right interval and ensure the right beliefs.

When the tax rate is the same for two or more intervals, cheap talk serves to transmit some information. In the terminology of Austen-Smith and Banks (2000), cost-free signalling is influential if two different cheap talk messages associated with the same tax rate have to be used to distinguish two different intervals.

Similarities with CS are obvious: Propositions 2 and 3 show that the government can use meaningful yet imprecise policies to communicate the side effects to consumers. This trade-off is classical for readers accustomed to cheap talk: the partition $\{I_k\}_{k \in K}$ entails a loss in precision, but now, if the government wants to lie, it has to pretend that the side effects are in a different subinterval, which changes consumers' consumption by a discrete amount. Such "big lies" are less attractive, in equilibrium, than telling the truth.

A less evident conclusion is that there are also considerable differences from CS. The skeleton approach enables us to show that partitions need not be finite, meaning that the precision of the message may be arbitrarily high locally (equilibria that fully reveal the information locally or globally exist, as we shall see). We retrieve the typical trade-off of signaling models: as precision increases, tax policies are more severely constrained by incentive compatibility, and distortions away from the second-best become large.

6 Partially Revealing Equilibria

Implementing a Skeleton We can start to build an equilibrium by choosing a skeleton and filling the distribution while preserving conditional expectations. Proceeding in this way, we can readily provide examples in which the tax rate is not monotonic, where it is revealing on certain subsets of the type support with bundles elsewhere, etc. Hence a multiplicity of partially revealing and pooling equilibria are conceivable.

Proposition 3 does not claim that some distribution F always exists. Indeed, even off-equilibrium beliefs are constrained to be in the type support, and we may be short of sufficiently dissuasive off-equilibrium beliefs to support a skeleton. We are nevertheless able to furnish a simple way of extending a skeleton to make F non empty, in other words, to implement the skeleton in a PBE of a certain economy. This has no equivalent in the literature, to our knowledge.

Proposition 4 If the type support is large enough, any skeleton is either directly implementable or can be made implementable by adding one policy (one belief and its associated tax).

Proof. Let us take an un-implementable skeleton. If $\lambda > 0$, the simplest way to complete it is to add a sufficiently low type, say $\mu_{min} < \mu_1$, coupled with $t_{SB}(\mu_{min})$. To check incentive compatibility within the completed skeleton, note that if belief μ_{min} is sufficiently small compared to μ_1, it is necessarily too small compared to any type of government drawn in $\{\mu_k\}_{k \in K}$. Moreover $t = t_{SB}(\mu_{min})$ is better than any other value of the tax in the skeleton for a government of type μ_{min}. Notice that, if μ_{min} is sufficiently low, by associating belief μ_{min} with any tax outside the completed skeleton $\{t_k\}_{k \in K}$, the completed skeleton can be supported as an equilibrium. As a consequence the completed skeleton is implementable.

This procedure implies that the type support $[\underline{\mu}, \overline{\mu}]$ has to be large enough: μ_{min} must be inside the type support. If $\lambda < 0$, the same reasoning holds for a large $\mu_{max} > \mu_1$ (coupled with $t_{SB}(\mu_{max})$). ■

Notice that the probability mass associated with the subinterval defined by the new point (μ_{min}, t_{SB}) can be arbitrarily low, as Proposition 3 has shown. This implies that extreme types can be extremely unlikely, but have important effects: offering sufficiently dissuasive off-equilibrium beliefs, they increase the set of implementable skeletons. Indeed, such "scarecrow" types may have either a negative or a positive impact on social welfare depending on the efficiency characterizing the tax policy they contribute to implement. If an inefficient allocation is implemented, type μ_{min} represents a sort of paranoid (but rational) belief which "paralyzes" the economy. Thus, the government and consumers would be better off if the type support could be reduced: if type μ_{min} is "pushed" out of the support, then the inefficient allocation is not sustainable any more as an equilibrium. The opposite holds for efficient skeletons. In this case, type μ_{min} represents a very improbable but useful value of side effects which "disciplines" the government. In such a situation reducing uncertainty would be negative: μ_{min} must reside inside the type support, even if consumers' priors may attribute to it an arbitrarily low probability.

To be rigorous and complete, this analysis should consider the full economic value of a test (say scientific research) that can exclude extreme types ex ante (i.e. that is before the government learns its type). Indeed, it could happen, by definition, that an extreme value is the true value. Given that this happens with small probability, the reasoning above underlines the dominant effect: the test can either have a positive or a negative value, depending on the continuation of the game.

A multiplicity of partially revealing and pooling equilibria are conceivable and we discussed the implementability of more or less efficient equilibria. Anyway we don't explicitly address the problem of selection among equilibria, since applying refinements in this model is a difficult task given the non-monotonicity of the government preferences. In compensation, our approach allows us to understand the priors necessitated to support a given (or desired) equilibrium allocation.

The Role of Cheap Talk The following remark explains the role of cheap talk in our model. With finite skeletons, cheap talk is almost useless. In fact, either all taxes are different or some are identical and we can use the continuity of the incentive constraints to modify the skeleton slightly and make all tax rates different. In other words suppose that $(m_k; t_k)$ and $(m_{k'}; t_{k'})$ are two equilibrium policies with $t_k = t_{k'} = t$ and $m_k \neq m_{k'}$ (cheap talk is useful). If we change one of the two taxes, the partition in the skeleton has to be modified, but changes remain small because there are only a finite number of bones (hence a finite number of continuous incentive constraints) and the welfare cost of changing is arbitrarily low. After this small tax change it is $t_k \neq t_{k'}$ and all information is again transmitted by the tax: cheap talk is useless.

This argument uses finite skeletons to give the intuition. A complete proof of this type of results of this type is in Manelli (1996) who shows that cheap talk closes but does not substantially extend the set of equilibrium allocations. His result confirms that the costly message is almost sufficient alone to convey information in our model. If we see taxes as a proxy for the various imperfect and costly instruments that a government can impose to affect cigarette consumption, a practical conclusion is that they are typically more efficient, in terms of the precision of information conveyed, than words.

7 Fully Revealing Equilibria

In most signaling studies, fully informative equilibria receive a particular treatment: they are characterizable, which is useful for exposing some of the trade-off one expects in any equilibrium. Fully revealing equilibria are allocations where all types are represented in the skeleton: all intervals of the type support's partition correspond with singletons. Moreover, we can see that a fully revealing allocation is a universal skeleton, that is an equilibrium for any distribution F in $[\underline{\mu}; \overline{\mu}]$ (see Proposition 3). The following proposition establishes that for any given $[\underline{\mu}; \overline{\mu}]$ there is a unique fully revealing equilibrium (or a unique universal skeleton), which we characterize in detail. The result is not an application of Mailath

(1987) since signals have non-monotonic costs. The proof has to be written in full to take care of this complication.

Proposition 5 There exists a unique fully revealing equilibrium. The tax rate is the unique solution to the ordinary differential equation $\frac{t'}{1+\lambda} = -\frac{t-\bar{t}}{t-t_{SB}(\theta)}$ with the boundary condition $t(\bar{\theta}) = t_{SB}(\bar{\theta})$ if $\lambda > 0$; and $t(\underline{\theta}) = t_{SB}(\underline{\theta})$ if $\lambda < 0$. In particular:

1. The strategy $t(\cdot)$ is strictly increasing and differentiable, and cheap talk is useless.
2. Consumption decreases with respect to θ.
3. If $\lambda > 0$; the tax rate exhibits no distortion at $\bar{\theta}$. For other values, the tax rate is lower than the second-best tax rate and higher than $\bar{t}$.
4. If $\lambda < 0$; the tax rate exhibits no distortion at $\underline{\theta}$. For other values the tax rate is higher than the second-best tax rate and lower than $\bar{t}$.

Proof. See the Appendix. ■

It is clear from point 1 that in the fully informative equilibrium all the information is transmitted through the tax rate, and cheap talk is useless. When $\lambda > 0$ ($\lambda < 0$); the fully informative equilibrium allocation, compared to the second-best one, is characterized by taxes that are too low (or too high). In any case taxes are increasing with respect to the type.

The solution to the differential equation lies between the optimal tax $t_{SB}(\theta)$ and the committed honesty tax $\bar{t}$.[18] Indeed, when the government is close to the optimum, incentives to lie are strong and the slope of the revealing tax schedule is very steep. On the other hand, as the tax rates approach the suboptimal $\bar{t}$; incentives to manipulate beliefs vanish, and the revealing tax schedule flattens.

Figure 3 shows the fully revealing tax rate for $\tau = 1$; $\underline{\theta} = 0$; $\bar{\theta} = 1$ and $\lambda = .3$. Notice the indifference curves passing through the equilibrium value for $\theta = .8$ and $\theta = 1$. Figure 4 corresponds to $\lambda = -.3$ (other parameters are equal to those in Figure 3). This illustrates the non-negligible size of the distortion.

Insert Figure 3 here.
Insert Figure 4 here.

The limited role of cheap talk can also be viewed in another way. As $\lambda \to 0$; one can find a sequence of fully-revealing equilibrium allocations converging on the first-best where $t = \tau$ for all θ. At the limit, cheap talk is indispensable, but very close approximations in which it is not used are available.

[18] On the properties of $\bar{t}$; see Section 4.

In CS, the most informative equilibrium Pareto-dominates, the others ex ante.[19] With our skeleton approach, it is relatively plain that the unique-fully informative equilibrium allocation need not be efficient. To see this, take an equilibrium and take its skeleton. The substance of Proposition 3 is that any economy that satisfies the restrictions on the conditional expected type in the intervals associated with the skeleton can implement it in equilibrium. If the mass of an interval where the distortion is substantial is sufficiently large, then the equilibrium is necessarily inefficient ex ante. More generally, given two skeletons, one more informative than the other (a finer partition in intervals), the less informative one can be made more efficient by choosing the distribution appropriately. This was noticed early in the signaling literature (and also in a previous version of Austen-Smith and Banks 2000), confirming that a model mixing signals and cheap talk is closer to models only composed of the former.

8 Conclusion

We have examined the conflict between providing incentives and transmitting information that arises when an informed and benevolent government combines linear taxes and information campaigns. Our model suggests that the government may have trouble gaining credibility for its actions and messages, even though its objective is aligned with the consumers'. The problem is that the government cannot commit to reveal information truthfully. Its instruments being imperfect, it has strong incentives to enhance their impact with providing biased information.

Short phrases whose content is too vague to be verifiable ("smoking is harmful to health") are often of very limited efficacy, and a costly tax is taken more seriously than mere propaganda. This result is in line with the empirical evidence of Bardsley and Olekalns (1999) on the impact of health warnings on cigarette packs. Over the past 35 years, they show, price (including tobacco taxes), real income, and demographic effects explain most of the variation in tobacco consumption, whereas health warnings on cigarette packs have had a relatively minor impact.

Tobacco taxes are a mere example. In general, depending on the kind of distortion that prevails in the fiscal system (i.e. whether the tax generates a "double dividend" or not), the government would like to make consumers either pessimistic or optimistic about the effect of consumption on individuals. In the likely case of positive marginal cost of public funds, if consumers were more optimistic, the government would see tax revenue increase, and the distortions created by preexisting taxes would be easily alleviated. Fuel taxes are another example: the government may not wish to stress the polluting effects or the

[19] See Theorems 3 and 5 in CS, which say that both the sender and the receiver strictly prefer equilibrium partition with more steps.

dangers of motor vehicles in order to preserve this ready source of revenue. The consequence is that, at the fully revealing equilibrium, there is a bias towards excessively low taxes. The structure by partitions of less informative equilibria gives credit to the possibility that there exist "psychological" thresholds in the tax level that cause discrete changes in the public's perception.

Examples of negative costs of public funds are quite informal, they concern antibiotics in public health insurance systems and SO_2 emissions in France. Antibiotics and SO_2 policies are implemented by independent agencies that choose, presumably, a policy that maximizes social welfare.[20] Thus, independent agencies have they own budgetary constraints and financing instruments: reimbursement for antibiotics expenditures, collecting the SO_2 tax, or setting controls, all these measures have an administrative cost that we can model with $\lambda < 0$. Consequently, it is tempting for the Social Security to exaggerate side effects of antibiotics. This can be done by limiting reimbursement or by developing controls to signal problematic drugs. As for SO_2 emissions, the agency is interested in exaggerating the polluting effects (i.e. the firm's perception of the compensation they could be asked in lawsuit). As a result, at the fully revealing equilibrium, the two agencies are biased towards too high taxes.

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A Appendix

A.1 Proof of Proposition 5

We establish the result in two steps. The first analyzes differentiable fully-revealing equilibria; uniqueness in this category is proved. The second step shows that any fully-revealing equilibrium is essentially identical to the differentiable one.

A.1.1 Differentiable Equilibrium

The analysis follows this plan: reasoning on local incentive compatibility, we find the ordinary differential equation satisfied by any fully-revealing equilibrium tax policy and we eliminate solutions with tax rates that do not fall between $\frac{\delta}{1+\lambda}$ and the second-best schedule (whichever is the higher); we check global incentive compatibility along the equilibrium policy; we search for off-equilibrium beliefs (i.e. associated with off-equilibrium tax rates) that discourage deviations. This gives a unique equilibrium.

Local Incentive Compatibility The government prefers $t(\theta)$ (and the implied $x(\theta)$) to $t(\theta + d\theta)$ and to $t(\theta - d\theta)$; taking limits we get

$$x'U' + \lambda t'x - (\delta + \theta - \lambda t)x' = 0. \tag{19}$$

Given that the consumer's first-order condition is

$$U' = \theta + t; \tag{20}$$

we can eliminate U' to get (after simplification)

$$tx' = -\lambda t'x + (\delta - \lambda t)x'. \tag{21}$$

t and x being separable, (21) is easily integrated to give

$$\forall\theta; \theta_0 : \frac{\frac{\delta}{1+\lambda} - t(\theta)}{\frac{\delta}{1+\lambda} - t(\theta_0)} = \left(\frac{x(\theta)}{x(\theta_0)}\right)^{\frac{1+\lambda}{\lambda}} \tag{22}$$

where θ_0 and $t_0 = t(\theta_0)$ are initial conditions. Equations (22) and (5) implicitly but entirely determine the solutions $t(\theta)$ and $x(\theta)$: In particular, x solves

$$\left((\theta + \frac{\delta}{1+\lambda})x - 1\right)x^{\frac{1}{\lambda}} = \left((\theta_0 + \frac{\delta}{1+\lambda})x_0 - 1\right)x_0^{\frac{1}{\lambda}} = \text{Constant} \tag{23}$$

By differentiation, we get

$$x' = \frac{\lambda x^2}{1 - (\delta + (1+\lambda)\theta)x} \tag{24}$$

The second-order condition is:

(25) $$0 \geq x'^2U'' + x''U' + \lambda t''x + 2\lambda t'x' - (\eta + \theta - \lambda t)x'';$$

while the derivative of the first-order condition is:

(26) $$0 = x'^2U'' + x''U' + \lambda t''x + 2\lambda t'x' - x' - (\eta + \theta - \lambda t)x'';$$

Simplifying (25) with (26) we get:

(27) $$x' \leq 0$$

Applying (27) to (24) and using (5), we can see that, when $\lambda > 0$; $x' \leq 0$ if and only if $t < \eta + \lambda\theta$ and when $\lambda < 0$; $x' \leq 0$ if and only if $t > \eta + \lambda\theta$:

Starting from (24) and (21), straightforward calculations prove that the differential equation satisfied by t is

(28) $$t' = \frac{\eta - (1+\lambda)t}{t - \eta - \lambda\theta}:$$

Global Incentive Compatibility The first- and second-order conditions exclude infinitesimal deviations. We check that discrete deviations are also precluded.

Let θ be the true value of the side-effects parameter. Using (5) we calculate the derivative of the government's utility with respect to $\hat{\theta}$ assuming that the government offers $t(\hat{\theta})$, thereby inducing $x(\hat{\theta})$:

(29) $$\begin{aligned} & x'(\hat{\theta})U'[x(\hat{\theta})] + (\lambda t'(\hat{\theta})x(\hat{\theta}) - (\eta + \theta - \lambda t(\hat{\theta}))x'(\hat{\theta})) \\ = \; & \frac{x'(\hat{\theta})}{x(\hat{\theta})} + \lambda t'(\hat{\theta})x(\hat{\theta}) - (\eta + \theta - \lambda t(\hat{\theta}))x'(\hat{\theta}) \end{aligned}$$

>From (21), we find that the following expression has the same sign as (29)

(30) $$x(\hat{\theta})(t(\hat{\theta}) + \theta) - 1$$

Using (5), it follows that (29) is positive for $\hat{\theta} < \theta$ and negative for $\hat{\theta} > \theta$: This means that incentive compatibility is satisfied everywhere for equilibrium actions.

Uniqueness of the Differentiable Equilibrium We give the full reasoning for $\lambda > 0$: A symmetrical argument proves the proposition for $\lambda < 0$: We show now that the boundary condition $t(\bar{\theta}) = \eta + \lambda\bar{\theta}$ is necessary.

Reasoning by contradiction, we show that there exist beliefs compatible with the equilibrium for off-equilibrium actions if and only if the condition is satisfied.

Let $t(¢)$ be a solution to (28) such that $t(\bar{\mu}) < ´ + ¸\bar{\mu}$:[21] Given (28), either $t(\mu)$ is systematically below $\frac{´}{1+¸}$ or $t(\mu)$ is strictly increasing. In any case $\max_\mu t(\mu) < ´ + ¸\bar{\mu}$; we choose an arbitrary (o¤-equilibrium) t in the interval $(\max_\mu t(\mu); ´ + ¸\bar{\mu})$ and we denote by $\hat{\mu}$ the associated belief. Now we prove that there always exists a type μ such that the government prefers policy $(\hat{\mu}; t)$ to policy $(\mu; t(\mu))$:

>From Proposition 1, we know that for each μ; the absolute best policy is $\frac{´+\mu}{¸}$ for the tax rate and $¡\frac{´+\mu}{¸}$ for the belief; moreover, the second-best $(\mu; ´ + ¸\mu)$ is preferred to $(\mu; t(\mu))$: The convexity of the upper contours of the government's objective function implies that any policy in the triangle $4(\mu) = ((\mu; t(\mu)); (\mu; ´ + ¸\mu); (¡\frac{´+\mu}{¸}; \frac{´+\mu}{¸}))$; except $(\mu; t(\mu))$, is strictly better than $(\mu; t(\mu))$ when μ is the type. It su¢ces now to check that $(\hat{\mu}; t)$ is necessarily in $4(\mu)$ for a certain μ 2 $[\underline{\mu}; \bar{\mu}]$: Indeed, $[_{\mu 2[\underline{\mu};\bar{\mu}]} 4(\mu)$ contains (a) the triangle $((\underline{\mu}; ´ + ¸\underline{\mu}); (\bar{\mu}; ´ + ¸\bar{\mu}); (¡\frac{´+\bar{\mu}}{¸}; \frac{´+\bar{\mu}}{¸}))$; and (b) the policies between $(\mu; t(\mu))$ and $(\mu; t_{SB}(\mu))$ for μ 2 $[\underline{\mu}; \bar{\mu}]$: Provided $\frac{´+\bar{\mu}}{¸}$ is larger than $´ + ¸\bar{\mu}$; then $(\hat{\mu}; t)$ is either in (a) or (b) in the latter union, hence the existence of a μ for which the deviation is desirable. Given that $\frac{´+\bar{\mu}}{¸} > ´ + ¸\bar{\mu}$; the proof is complete. The consequence is that $t(\bar{\mu}) = ´ + ¸\bar{\mu}$ (no distortion at the top).

Now we prove that associating belief $\bar{\mu}$ to any tax rate above $t(\bar{\mu})$ does not induce deviations. The value to the government of type μ of imposing $t > t(\bar{\mu})$; thereby inducing belief $\bar{\mu}$; is: $¡ \log(\bar{\mu} + t) ¡ \frac{´+\mu ¡ ¸t}{\bar{\mu}+t}$: The root of the derivative with respect to t is $´ + \mu ¡ (1 ¡ ¸)\bar{\mu}$ which is lower than $´ + ¸\bar{\mu} = t(\bar{\mu})$: The value being decreasing with respect to t over $[t(\bar{\mu}); +1[$, $t(\bar{\mu})$ is a better move than any $t > t(\bar{\mu})$: Given that equilibrium actions are incentive-compatible, neither $t(\bar{\mu})$ nor t is desirable, compared to $t(\mu)$. By the same reasoning, we can check that, if for $t < t(\underline{\mu})$; beliefs are $\underline{\mu}$; then t is not attractive: for a government of type μ; the value of imposing $t < t(\underline{\mu})$ is smaller than that of imposing $(\underline{\mu}; t(\underline{\mu}))$:

We conclude that the unique revealing allocation found is an equilibrium.

A.1.2 Uniqueness in General

Let us take a fully-revealing equilibrium. Given that the government's preferences, for constant beliefs, are single-peaked with respect to t (a direct consequence of the convexity in Proposition 1), and given the value of its equilibrium strategy, there exist a maximum of two tax rates per μ; $t_L(\mu)$ and $t_U(\mu)$; both being suboptimal (as compared to the second-best) when they are di¤erent. More precisely, $t_L(\mu)$ **6** $´ + ¸\mu$ **6** $t_U(\mu))$. The theorem of the maximum ensures that the value of the government's equilibrium strategy is continuous with respect to μ; therefore functions $t_L(¢)$ and $t_U(¢)$ are continuous with respect to μ: We denote by $£_L$ and $£_U$ the subsets of $[\underline{\mu}; \bar{\mu}]$ leading to a move in the lower and the upper

[21] For $¸ > 0$; we already excluded that the tax rate could be higher than the second-best tax rate in the preceding subsection.

selection respectively. Notice that $\Theta_L \cup \Theta_U = [\underline{\theta};\overline{\theta}]$ but $\Theta_L \cap \Theta_U \neq \emptyset$ if mixed strategies are used. For fixing ideas, the following reasoning assumes that $\lambda > 0$.

The first step is to prove that Θ_U is not dense in any interval of $[\underline{\theta};\overline{\theta}]$. We reason by contradiction: take J an interval in $[\underline{\theta};\overline{\theta}]$ in which Θ_U is dense. Take $\theta_0 \in J$, and a strictly monotonic sequence $(\theta_n)_{n\geq 1}$ in Θ_U converging to θ_0. We prove that for all sequences $(\theta_n)_{n\geq 1}$, $\lim_{n\to\infty} \frac{t_n - t_0}{\theta_n - \theta_0} = \frac{\eta - (1+\lambda)t_0}{t_0 - \eta + \lambda\theta_0}$, where t_n denotes $t_U(\theta_n)$. Indeed, incentive constraints (θ_n should not mimic θ_0, and vice-versa) imply that:

$$(31)\qquad -\log(\theta_n + t_n) - \frac{\eta + \theta_n - \lambda t_n}{\theta_n + t_n} \geq -\log(\theta_0 + t_0) - \frac{\eta + \theta_n - \lambda t_0}{\theta_0 + t_0}$$

$$(32)\qquad -\log(\theta_n + t_n) - \frac{\eta + \theta_0 - \lambda t_n}{\theta_n + t_n} \leq -\log(\theta_0 + t_0) - \frac{\eta + \theta_0 - \lambda t_0}{\theta_0 + t_0}$$

Therefore, taking a first-order approximation, and multiplying by $(\theta_0 + t_0)^2$ yields

$$(33)\qquad 0 \geq ((1+\lambda)t - \eta)(\theta_n - \theta_0) + (t - \eta - \lambda\theta)(t_n - t_0) + o(\theta_n - \theta_0) + o(t_n - t_0)$$

$$(34)\qquad 0 \leq ((1+\lambda)t - \eta)(\theta_n - \theta_0) + (t - \eta - \lambda\theta)(t_n - t_0) + o(\theta_n - \theta_0) + o(t_n - t_0)$$

The limit of the rate of variations is the same for all sequences, which implies that t_U is differentiable at θ_0, hence differentiable on interval J.

A solution of the differential equation (28) situated above the second-best taxes is not incentive compatible at any point, because the second-order condition is never satisfied. We can conclude that strategy t_U is not-incentive compatible, and that interval J does not exist.

It is now easy to conclude that Θ_L is dense in $[\underline{\theta};\overline{\theta}]$: being the complementary set (in an interval) of a set Θ_U which is not dense anywhere, Θ_L is dense. Consequently, t_L satisfies the differential equation (28) in a dense subset of $[\underline{\theta};\overline{\theta}]$, which implies that it does so everywhere. The lower selection is necessarily equal to the unique differentiable equilibrium strategy, since we can apply to $t_L(\cdot)$ the reasoning suited for differentiable equilibria.

It remains now to prove that Θ_U contains a finite number of points. We take θ_1 and $\theta_2 \in \Theta_U$ (where $\theta_1 \neq \theta_2$) with corresponding tax rates t_1 and t_2. Let us denote by $t_i(\cdot)$ $(i = 1; 2)$ the solution to (28) with maximal definition domain passing through t_i at θ_i. Note that either $t_1(\cdot)$ and $t_2(\cdot)$ are the same, or one is systematically above the other, because, according to the Cauchy-Lipschitz Theorem, two different solutions to differential equation (28) never cross.

Assume for fixing ideas that $t_2(\cdot)$ is above $t_1(\cdot)$. (a) If the two curves are close enough to each other, $t_2(\theta_1)$ is defined and is larger than t_1. Notice that $t_2(\theta_1)$ is closer to the second-best than t_1. Our study of the incentives when taxes are above the second-best shows that solutions to the differential equations minimize welfare (the second-order condition is violated everywhere): when the type is μ_1; t_2 with belief μ_2 is preferred to $t_2(\mu_1)$ with belief μ_1: By transitivity, t_2 is preferable to t_1 when the true type is μ_2: This is in contradiction with incentives. (b) If there is an in...nite number of types in $£_U$; we can always exhibit μ_1 and μ_2 which are close enough to each other to apply reasoning (a). We conclude that $£_U$ contains a ...nite number of points.

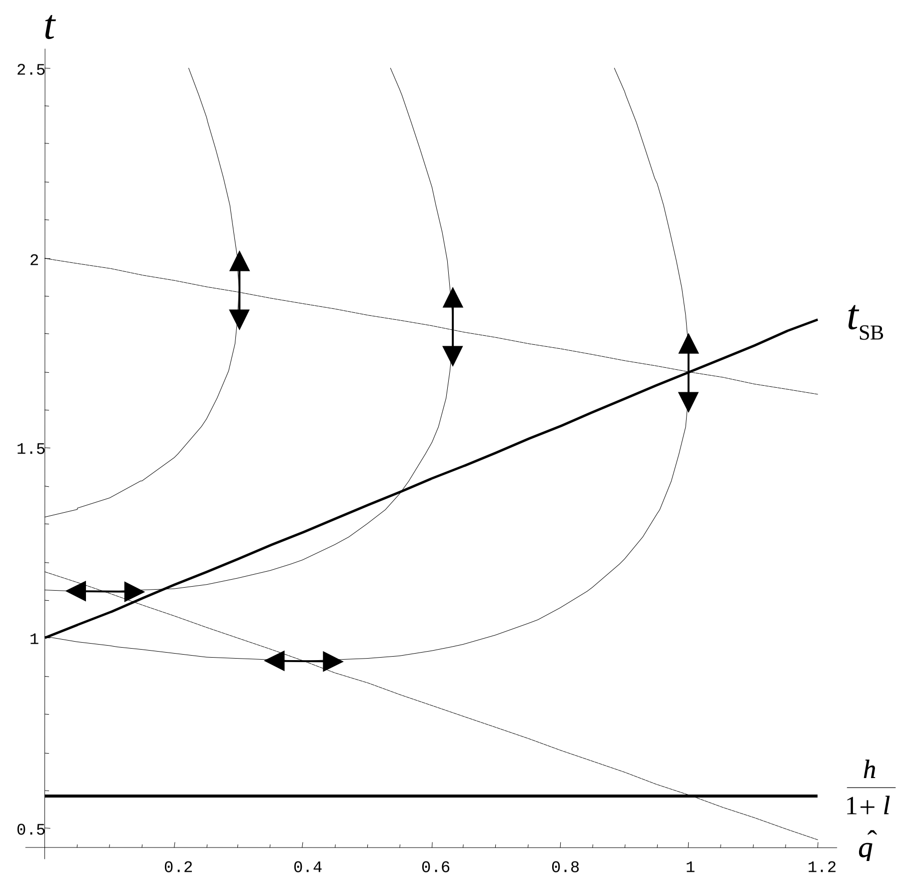

Figure 1

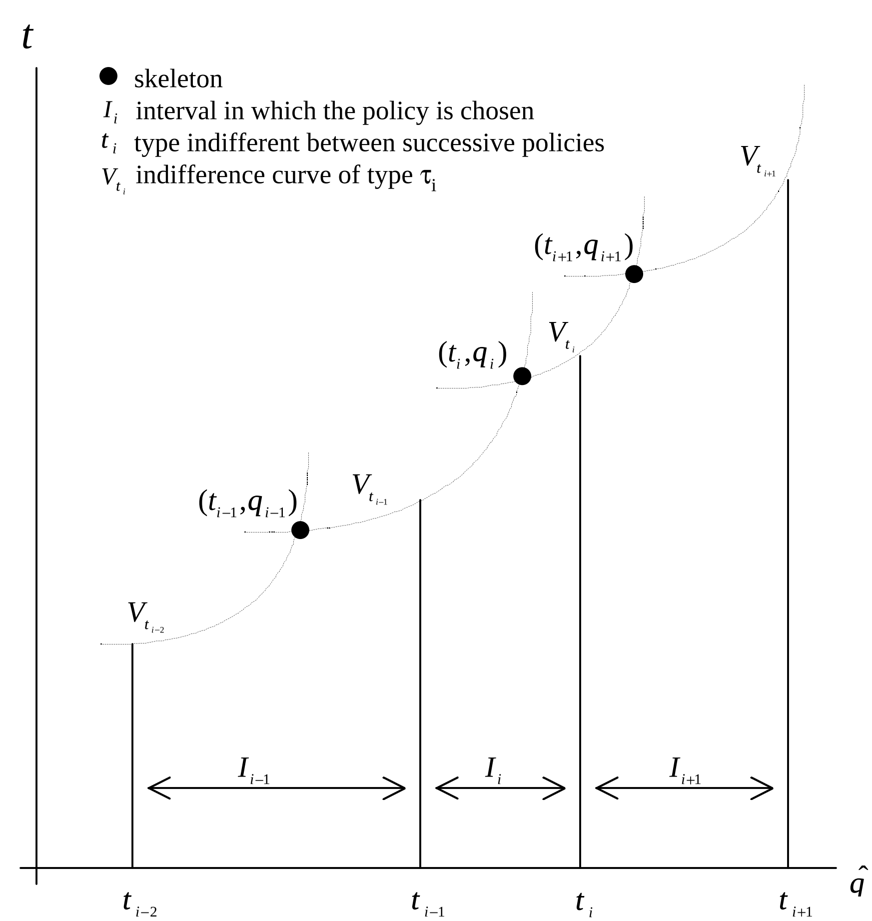
t
skeleton
I_i interval in which the policy is chosen
t_i type indifferent between successive policies
V_{t_i} indifference curve of type τ_i
$V_{t_{i+1}}$
(t_{i+1},q_{i+1})
(t_i,q_i)
V_{t_i}
(t_{i-1},q_{i-1})
$V_{t_{i-1}}$
$V_{t_{i-2}}$
I_{i-1}
I_i
I_{i+1}
$\hat{q}$
t_{i-2}
t_{i-1}
t_i
t_{i+1}

Figure 2

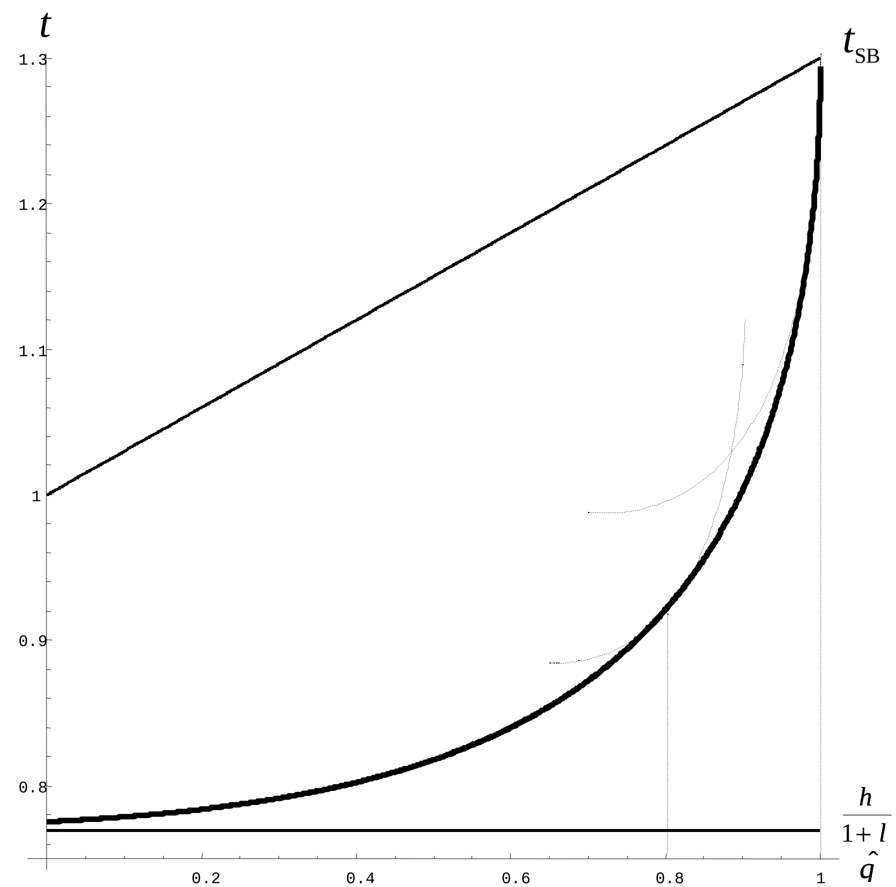

Figure 3

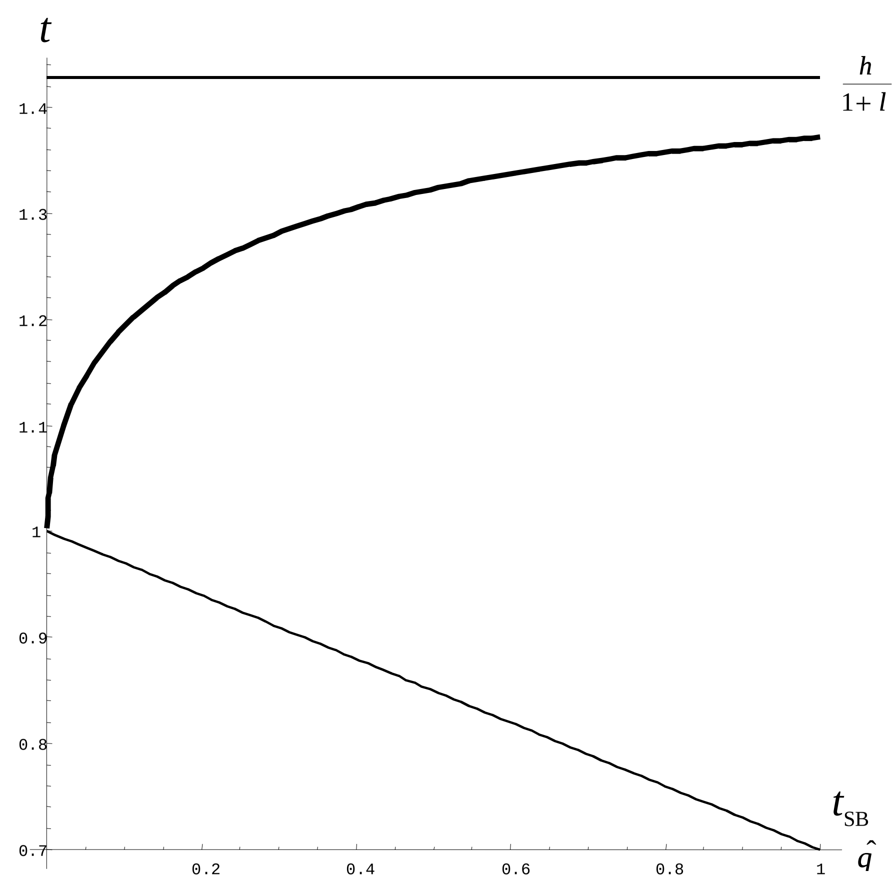

Figure 4