

A Model of Add-on Pricing

Glenn Ellison¹

Massachusetts Institute of Technology and NBER

May 2003

¹I would like to thank the National Science Foundation for financial support. This paper grew out of joint work with Sara Fisher Ellison. Joshua Fischman provided excellent research assistance. Jonathan Levin and Steve Tadelis provided helpful feedback.

Abstract

This paper examines a competitive model of add-on pricing, the practice of advertising low prices for one good in hopes of selling additional products (or a higher quality product) to consumers at a high price at the point of sale. The main conclusion is that add-on pricing softens price competition between firms and results in higher equilibrium profits.

JEL Classification No.: L13, M30

1 Introduction

In many businesses it is customary to advertise a base price for a product or service and to try to induce customers to buy additional “add-ons” at high prices at the point of sale. The quoted price for a hotel room may not include phone calls, in-room movies, minibar items, dry cleaning, etc. Electronics stores offer extended warranties and accessories. Car rental agencies offer insurance and prepaid gasoline. New car dealers hope to service cars they sell. Add-ons need not be a separate goods. For example, a high quality mattress can be thought of as a bundle containing a low quality mattress and an add-on of additional quality. Once one starts to think broadly about add-on pricing, it can be challenge to think of a business where the practice is absent.

Although many firms earn a substantial portion of their revenue from add-ons, it is not clear that we should care.¹ The classic Chicago school argument would be that profits earned on add-ons will be competed away in the form of lower prices for the base good. Two formalizations of this argument can be found in the literature. Lal and Matutes (1994) develops a model of loss-leader pricing in which the Chicago view is true to an extreme – every consumer purchases the same bundle at the same price regardless of whether the prices of add-ons are or are not advertised. Verboven (1999) analyzes a model of add-on pricing with different assumptions about preferences in which add-on pricing again has no effect on profits. This paper, in contrast, argues that add-on pricing is important. It notes that there is an adverse selection problem that arises when competing firms practice second degree price discrimination, and shows that as a result add-on pricing may soften competition and raise equilibrium profits.

The model of this paper is similar to the model of Lal and Matutes (1994), but with horizontal and vertical taste differences. Two firms are located at the opposite ends of a Hotelling line. Each firm has two products for sale: a base good and an add-on. The add-on provides additional utility if consumed with the base good. There are two continuums of consumers: “high types” with a low marginal utility of income; and “low types” or “cheap-skates” with a high marginal utility of income. Within each subpopulation, consumers have

¹CNNMoney reported that the credit card industry received \$7 billion in late payment fees in 2001. See <http://money.cnn.com/2002/05/21/pf/banking/cardfees/>.

have unit demands for the base good with the standard uniformly distributed idiosyncratic preference for buying from firm 1 or firm 2. As in Verboven (1999), the analysis consists primarily of an analysis of two games: a “standard” pricing game in which the firms simultaneously announce both a price for the base good and a price for a bundle containing the base good and the add-on; and an add-on pricing game in which the firms only announce the price for the base good and consumers do not learn the price of the add-on until they have incurred a sunk cost to visit the firm.

It is not hard to construct models in which add-on pricing is irrelevant: the simplest would be a price competition game where firms announce a price and then charge all consumers exactly \$17 more than the price they announced. Section 3 shows that a benchmark case of the model described above is another example. As long as the preferences of the high and low types are not too dissimilar the standard pricing game and the add-on pricing game have identical outcomes: every consumer makes the same purchase and pays the same price in the two games and the firms are therefore also equally well off.

Section 4 contains the main result of the paper. It shows that when the high and low types are more different, the firms’ profits are higher in the add-on pricing game than in the standard pricing game. My view is that this should not just be of interest to those who work on pricing and those who regard the mechanism as neat. To the contrary, I suspect that add-on pricing may be an important component of the correct explanation for how firms are able to survive in world with substantial fixed costs. In many of the examples given in the first paragraph, e.g. hotels, car rental agencies, and retail stores, prices must be substantially above marginal costs to recover fixed costs. Although the industrial organization literature has a number of standard explanations for how marginal cost pricing can be avoided, such as product differentiation, search costs, and dynamic collusion, it is not clear that they can account for observed markups. Do consumers really have strong preferences for getting a Dollar rental car over a National rental car, for staying at the Hyatt instead of the Marriott, or for shopping at Sears instead of K-mart? The competition-softening effect of add-on pricing may be an important additional factor.

Comparing the add-on pricing game with the standard pricing game, I find that low types pay less in the add-on pricing game, while the high types pay more. The consumer welfare implications are more stark when comparing the outcome of the add-on pricing

game with a with what would happen if firms could be prevented from selling the low quality good, e.g. if refueling charges at car rental agencies were limited to a reasonable per gallon price. In this comparison, *both* types of consumers are made worse off by the feasibility of add-on pricing.

One way to understand the main result is to think of it as a “strategic investment” story. When the low and high types’ marginal utilities of income are sufficiently different, selling the add-on to the high types at a high price is more profitable than selling the add-on to everyone at a low price. In equilibrium, of course, consumers are not fooled. The add-ons do, however, end up being more expensive than they would be if all prices were advertised, so add-on pricing can be seen as a strategic commitment to charge a high price for the add-on. Such a commitment will obviously affect equilibrium profits. Section 4 presents intuition for why the commitment increases profits.

Another way to think about the main result is to regard it as a story about firms intentionally creating an adverse selection problem in order to soften competition. In the health insurance market, adverse selection limits the completeness of insurance policies – if a firm were to offer a more complete policy, then it would attract a customer pool with disproportionate share of sick people. When customers are heterogeneous in their marginal utility of income, there is a similar selection effect: a firm that undercuts its rivals on price will attract a customer pool that contains a disproportionate share of cheapskates. If each firm sells a single good, this selection effect is not adverse – a cheapskate’s money is as good as anyone else’s. When firms offer multiple goods and add-on pricing policies keep the low- and high quality prices far apart, the selection becomes adverse – firms do not want to attract a large number of cheapskates who only buy the loss leader. The incentive to cut price is reduced, and hence equilibrium profits go up.

Sections 3 and 4 demonstrate that firms may be better off if they jointly choose to set prices and advertise as in the add-on pricing game. They do not address whether setting prices as in the add-on pricing game is individually rational in an expanded model where what is advertised when is endogenous. Section 5 notes that if one endogenizes the price-setting and advertising process in the simplest way, then adopting add-on pricing is not individually rational. It then discusses a number of natural variants of the model in which firms would individually choose to adopt add-on pricing policies.

Section 6 examines a variant of the model in which only a small fraction of the population are cheapskates. In this model adopting add-on pricing is a classic example of a competitive strategy that turns lemons into lemonade. It does not just mitigate the damage that cheapskates do to equilibrium profits; it creates an environment where firms benefit from the presence of cheapskates.

Section 7 relates the paper to the literatures on loss-leaders, competitive price discrimination, and other topics. Section 8 concludes.

2 Model

I consider a variant of the standard competition-on-a-line model with vertical as well as horizontal differentiation. There are two firms indexed by $i \in \{1, 2\}$. Each firm sells two vertically differentiated goods, L and H , and prices p_{iL} and p_{iH} . The firms can produce either L or H at a constant marginal cost of c .² Consumers differ in two dimensions. First, they differ in their marginal utility α of income. There are a unit mass of consumers with $\alpha = \alpha_h$ and a unit mass of consumers with $\alpha = \alpha_\ell$. We assume $\alpha_h < \alpha_\ell$. Thinking about their willingness to pay I will refer to group h as the “high” types and to group ℓ as the “low” or “cheapskate” types. Within each group customers are differentiated by a parameter $\theta \sim U[0, 1]$ that reflects how well the two firms’ products match their tastes.³ Assume that each consumer wishes to purchase at most one unit of one of the two products. Assume that a consumer receives zero utility if he or she does not make a purchase. If a consumer of type (α, θ) purchases exactly one unit his or her utility is

$$u(q_{1L}, q_{1H}, q_{2L}, q_{2H}; \alpha, \theta) = \begin{cases} v - \theta - \alpha p_{1H} & \text{if } q_{1H} = 1 \\ v - (1 - \theta) - \alpha p_{2H} & \text{if } q_{2H} = 1 \\ v - w - \theta - \alpha p_{1L} & \text{if } q_{1L} = 1 \\ v - w - (1 - \theta) - \alpha p_{2L} & \text{if } q_{2L} = 1 \end{cases}$$

Note the assumption of a lower marginal utility of income implies that the high types have a higher incremental valuation for high quality in money terms and are less sensitive to

²Good L can be thought of as a “damaged good” as in Deneckere and McAfee (1996).

³Note that I have fixed the range of the idiosyncratic taste parameter. To capture markets with only a small amount of horizontal differentiation, one would assume that α_ℓ and α_h are both large.

price differences between the firms. One could apply the model to any situation where this association makes sense even if it has nothing to do with differences in the marginal utility of wealth. For example, in the credit card market the low types could be wealthier, more sophisticated consumers who compare annual fees and interest rates more carefully when choosing between offers and who also are less likely to incur late payment fees.

Sections 3 and 4 will contrast the outcomes of two games: a standard price competition game in which the firms simultaneously post prices for both products; and an add-on pricing game where the firms post prices for good L and reveal their prices for good H only when consumers visit the firm. Consumers will, of course, have rational expectations about the nonposted prices. To model what happens if (out of equilibrium) these expectations turn out to be incorrect, I adopt a version of Diamond's search model where consumers incur a small sunk cost of s utils in visiting a firm. This cost must be incurred to purchase from a store or to learn its price for good H . Timelines for the standard pricing game and the add-on pricing game are shown below.⁴ The standard pricing game is similar, but with each firm choosing both prices at $t = 1$ and with consumers observing all prices.

In analyzing the model I will look at sequential equilibria. If the model were specified as a game between the firms with consumer behavior represented by demand functions, then it would be a complete information game in which one would require subgame perfection. With consumers as players in the game, however, one must deal with consumers' beliefs about the nonposted prices. The key restriction that sequential equilibrium places on these beliefs is that if a consumer visits firm 1 at $t = 3$ and learns that it has deviated from its equilibrium strategy, then the consumer continues to believe that firm 2's nonposted price is given by firm 2's equilibrium strategy. In the standard pricing game the sequential and subgame perfect equilibria coincide.

In the model all consumers will purchase either L or H in equilibrium if v is sufficiently large. Rather than letting this paper get cluttered with statements about how large v must be at various points, I will just make the blanket assumption here that v is sufficiently large so that all consumers are served in the relevant cases and not mention it again.

⁴The slightly odd-looking assumption that consumers can not visit a store at $t = 4$ if they have not visited a store at $t = 3$ is a device to rule out equilibria in which all consumers wait until $t = 4$ to shop and thereby lose the opportunity to switch stores if prices are not as they expect.

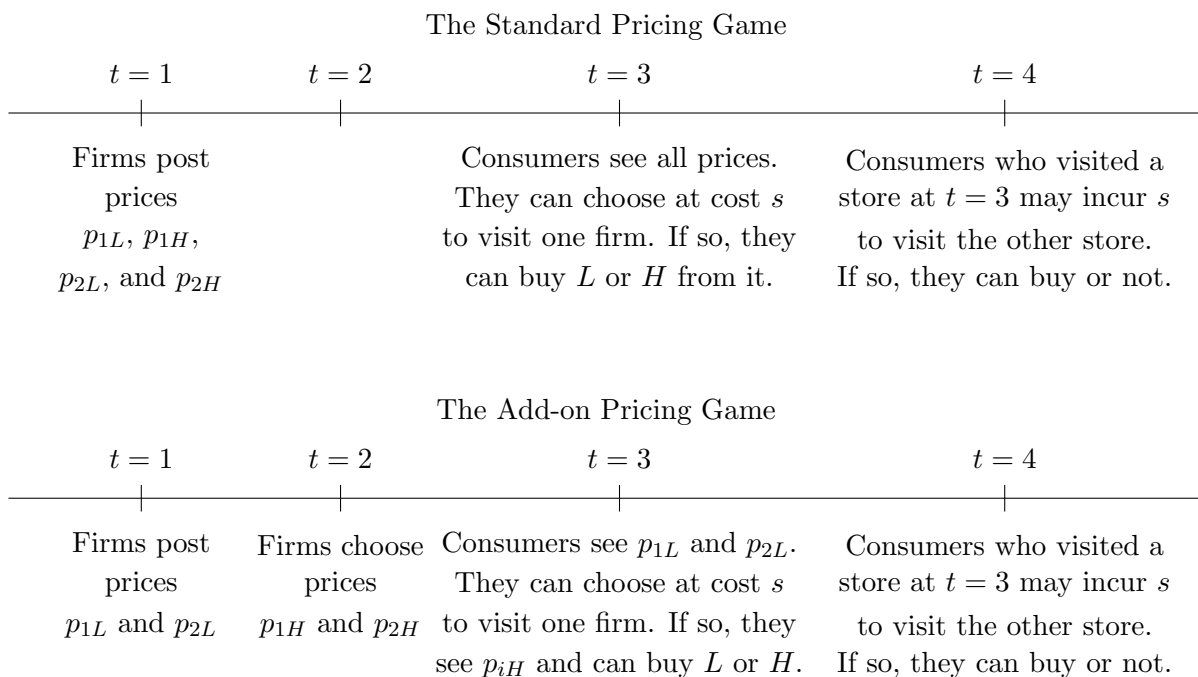


Figure 1: Timelines for the standard pricing and add-on pricing games

3 The Lal-Matutes benchmark: add-ons sold to everyone have no effect

Although Lal and Matutes (1994) is best-known for its conclusion that multi-product retailers may advertise a single good at a low price to save on advertising expenditures, it also contains an irrelevance result about loss-leader pricing – it shows that the bundle of goods each consumer purchases and the total amount each consumer pays are exactly the same with loss-leader pricing as they are when all prices were advertised.⁵ With no advertising costs this results in profits being equal as well. When $\alpha_h = \alpha_\ell$, the add-on pricing game of this paper is essentially the same as that of Lal and Matutes. In this section, I verify that the irrelevance result also carries over when α_h and α_ℓ are a bit different.

⁵The exact irrelevance result obviously requires special assumptions. Most notably, demands must be completely inelastic up to a cutoff point. I have chosen to make the same assumptions here both because it makes the model tractable and because it creates the contrast that highlights the competition-softening effect discussed in the next section.

Intuitively, the result should not be surprising. When α_ℓ and α_h are not too different, customers can forecast that they will be held up for the low type's valuation for the add-on once they visit the firm. Hence, it is little different from a game where instead of announcing their prices, firms announce a number that is exactly $1/\bar{\alpha}$ below their price. The argument is virtually identical to that of Lal and Matutes (and tedious) so I will not try to prove it under the weakest possible assumptions and will only sketch the argument in the text leaving the details to the appendix.

Proposition 1 *Suppose $\alpha_\ell/\alpha_h \leq 1.6$. Write $\bar{\alpha}$ for $(\alpha_\ell + \alpha_h)/2$. Then for v sufficiently large*

(a) *In any symmetric pure-strategy sequential equilibrium of the standard pricing game all consumers buy the high-quality good from the closest firm at a price of $c + 1/\bar{\alpha}$.*

(b) *In any symmetric pure-strategy sequential equilibrium of the add-on pricing game all consumers buy the high-quality good from the closest firm at a price of $c + 1/\bar{\alpha}$.*

Sketch of Proof

(a) In the standard pricing game, if all consumers buy H at a price of p_H^* , then if firm 1 deviates to a price p_{1H} in a neighborhood of p_H^* its profits are

$$\pi_1(p_{1H}) = \left(1 + \frac{\alpha_\ell + \alpha_h}{2}(p_H^* - p_{1H})\right)(p_{1H} - c)$$

A necessary condition for Nash equilibrium is that the derivative of this expression be zero at $p_{1H} = p_H^*$. This gives $p_H^* = \frac{1}{2}\left(c + \frac{1}{\bar{\alpha}} + p_H^*\right)$, which implies that any equilibrium of this form has $p_H^* = c + 1/\bar{\alpha}$.

The proof in the appendix verifies that the various possible nonlocal deviations also do not increase a firm's profits and hence that any profile where each firm's prices satisfy $p_{iH} = c + 1/\bar{\alpha}$ and $p_{iL} \geq c + 1/\bar{\alpha} - w/\alpha_\ell$ does yield an equilibrium.

The one alternate form of equilibrium that is not implausible is that the firms might sell good L to the low types and good H to the high types as part of a "damaged good" second-degree price discrimination strategy as in Deneckere and McAfee (1996). Damaged goods, however, are not always useful in price discrimination models. Good L is less valuable, but no less costly to produce. To get the low types to buy L instead of H , it must be offered at a

substantially lower markup. The appendix shows that for the parameter values considered here (with α_ℓ and α_h not too different) this makes the damaged good strategy nonviable.

(b) In the add-on pricing model, we can think of the firm i as advertising a price p_{iL} for good L at $t = 1$ and then choosing a nonposted price $p_{iU} \equiv p_{iH} - p_{iL}$ for an upgrade from L to H at $t = 2$. As in Diamond (1971), the fact that consumers search costs are sunk when they arrive at the firm ensures that the firms will set the monopoly price for the upgrade in equilibrium. When p_{1L} and p_{2L} are not too different and α_ℓ and α_h are sufficiently close together, a monopolist would choose to sell the upgrade to everyone at a price of w/α_ℓ . When p_{1L} is in a neighborhood of the symmetric equilibrium price p_L^* , consumers will correctly anticipate that if they visit firm j they will end up buying H at a price of $p_{jL} + w/\alpha_\ell$. Firm 1's profits are thus

$$\pi_1(p_{1L}) = \left(1 + \frac{\alpha_\ell + \alpha_h}{2}(p_L^* - p_{1L})\right) (p_{1L} + w/\alpha_\ell - c).$$

The FOC gives that the only possible equilibrium price is $p_L^* = c + 1/\bar{\alpha} - w/\alpha_\ell$.

The proof in the appendix again verifies that there is an equilibrium in which firms charge this price for the low-quality good and that there are no other symmetric pure-strategy equilibria.

QED

Note that although everyone buys good H at a price of $c + 1/\bar{\alpha}$, the price of good L is $c + 1/\bar{\alpha} - w/\alpha_\ell$. The proposition contains no restrictions on w , so this price can be below cost. Lal and Matutes (1994) describe their model as a model of loss leaders for this reason.

In Verboven's (1999) model consumers are horizontally and vertically differentiated and the complete irrelevance result of Lal and Matutes does not hold. Low types pay less in the add-on pricing game than in the standard pricing game and high types pay more. The profits part of the irrelevance result nonetheless carries over. The higher price paid by one group exactly offsets the lower prices paid by the other and the firms's profits are identical in the two games.⁶

⁶I thank Frank Verboven for this observation.

4 Discriminatory add-on pricing softens competition

In this section I present the paper's main observation – that add-on pricing softens competition and yields higher profits when add-ons are sold only to some consumers. Proposition 2 illustrates this observation by analyzing the behavior of the standard and add-on pricing game for a different set of parameter values.

One assumption is that the marginal utilities of income in the two populations are more different ($\alpha_\ell/\alpha_h > 3.2$). In the add-on pricing game, this makes firms want to sell the add-on to half of their customers at a price of w/α_h rather than to all of them at a price of w/α_ℓ . In the standard pricing game, firms sometimes sell good L as part of a “damaged goods” price discrimination strategy. The restriction on the size w of the add-on plays two roles: it simplifies the algebra and implies that by adopting add-on pricing firms, have essentially committed themselves to keeping p_L and p_H farther apart than they would be in the equilibrium of the standard pricing game. This is the strategic commitment that softens competition.

Proposition 2 *Suppose $\alpha_\ell/\alpha_h \in [3.2, 10]$. Let $\underline{w} = \alpha_h \left(\frac{1}{\alpha_h} - \frac{1}{\alpha_\ell} \right)$. Let $\bar{w} = 4 \left(\frac{\bar{\alpha}}{\sqrt{\alpha_\ell \alpha_h}} - 1 \right)$. Then $\bar{w} > \underline{w}$ and for $w \in (\underline{w}, \bar{w})$,*

(a) *The standard pricing game has a “discriminatory” sequential equilibrium in which the low types buy good L from the closest firm at a price of $c + 1/\alpha_\ell$ and the high types buy good H from the closest firm at a price of $c + 1/\alpha_h$. For some parameter values there is also a sequential equilibrium in which all consumers buy good H from the closest firm at a price of $c + 1/\bar{\alpha}$. There are no other symmetric pure-strategy equilibria.*

(b) *The add-on pricing game has a sequential equilibrium in which the firms set $p_{iL} = c + 1/\bar{\alpha} - w/2\bar{\alpha}$, low types buy good L from the closest firm, and high types pay w/α_h more to upgrade to good H . This is the only symmetric pure strategy equilibrium in which the equilibrium played at $t = 2$ is always that which is optimal for the firms.*

The firms' profits in the equilibrium of the add-on pricing game described above are $(w - \underline{w}) \frac{\alpha_\ell \alpha_h}{4\bar{\alpha} \alpha_h}$ greater than their profits in the discriminatory equilibrium of the standard pricing game. Profits are even lower in the nondiscriminatory equilibrium of the standard pricing game.

Sketch of Proof

(a) When the firms choose $p_{iL} = c + 1/\alpha_\ell$ and $p_{iH} = c + 1/\alpha_h$ in the standard pricing game, high types will buy good H rather than good L because $\alpha_h(p_{iH} - p_{iL}) = \alpha_h\left(\frac{1}{\alpha_h} - \frac{1}{\alpha_\ell}\right) = \underline{w} < w$. After some algebra one can also see that the $w < \bar{w}$ condition is sufficient to ensure that low types prefer L to H . For small deviations in price it is as if the firms were playing two separate competition-on-a-line games: one involving selling good L to low types and one involving selling good H to high types. The standard calculations for these games show that a small change in p_{1L} or p_{1H} will not increase firm 1's profits.

Completing the proof that this is an equilibrium requires showing that firm 1 also cannot increase its profits by selling H to members of both populations. When w is large enough such a deviation is profitable – good L is sufficiently damaged so as to make the benefits from selling the low types a better product outweigh the price discrimination benefits of selling L . The upper bound \bar{w} was chosen to ensure that a deviation that involves selling only H is not profitable. The appendix contains this calculation along with other details of the argument above.

The appendix also shows that the nondiscriminatory equilibrium of Proposition 1 remains an equilibrium for a proper subset of the parameter values covered by Proposition 2 (those with α_ℓ/α_h or w large) and that there are no other symmetric pure strategy equilibria.

(b) In the add-on pricing game the lower bound on α_ℓ/α_h ensures that when p_{1L} and p_{2L} are close together, the best equilibrium (for the firms) has both firms pricing the add-on at $p_{iU} = w/\alpha_h$ at $t = 2$. Firm 1's profit function (for small deviations) is thus

$$\pi_1(p_{1L}, p_{2L}) = \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_{2L} - p_{1L})\right)(p_{1L} - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_{2L} - p_{1L})\right)(p_{1L} + w/\alpha_h - c)$$

Considering the first-order conditions for firm 1's profit maximization shows that $p_{iL} = c + 1/\bar{\alpha} - w/2\bar{\alpha}$ is the only possible first period price in a symmetric pure-strategy equilibrium. This profit function is concave, so no price p_{1L} for which the profit function applies can increase firm 1's profits. It remains only to show that firm 1 cannot increase its profits via a larger deviation, for example with a larger reduction in price that will let it sell to all of the low types (which yields a higher profit than the above formula gives when p_{1L} is below cost). The assumption that $\alpha_\ell/\alpha_h < 10$ in the proposition is a convenient way to ensure

that the profile is indeed an equilibrium. (Weaker conditions could be given with some more work.) Again, the details are in the appendix.

In the discriminatory equilibrium of the standard pricing game, each firm's profit is

$$\pi^{s,d} = \frac{1}{2} \left(\frac{1}{\alpha_\ell} + \frac{1}{\alpha_h} \right).$$

In the specified equilibrium of the add-on pricing game, each firm's profit is

$$\pi^a = \frac{1}{2} \left(\frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} \right) + \frac{1}{2} \left(\frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} + \frac{w}{\alpha_h} \right)$$

Simplifying the difference between these two expressions gives the result on the difference in profits. The profits in the nondiscriminatory equilibrium of the standard pricing game are $1/\bar{\alpha}$. This is less than $\pi^{s,d}$ by the standard inequality on arithmetic and harmonic means.

QED

Remarks:

1. The view of add-on pricing that consumers should have in light of the equilibrium effects of add-on pricing described in Propositions 1 and 2 is counter to what one often hears from consumer groups. For example, there was great popular uproar when, in the midst of the electricity crisis of 2001, some hotel chains started adding a fixed daily energy surcharge to every bill. Proposition 1 suggests that consumers should not dislike the practice. If all consumers are aware of the practice it will have no effect on the total bill in equilibrium. High prices for minibar items and in-room movies are often seen as less outrageous because consumers can avoid paying the high prices by not consuming the add-ons. Proposition 2, however, indicates that it is precisely the voluntary nature of such consumption that leads to higher equilibrium prices.

2. Another natural point of comparison is how prices in the add-on pricing game differ from those that would prevail if the damaged good L did not exist. In this situation, the equilibrium price of good H would again be $c + 1/\bar{\alpha}$.⁷ Hence, the invention of good L increases the firms' profits. An interesting further observation is that in contrast to how we normally think about price discrimination, the invention of the damaged good and

⁷That this is an equilibrium and not just the solution to the first-order conditions follows from the analysis of the possibility of a uniform pricing equilibrium in the standard pricing game in the appendix.

the adoption of add-on pricing makes *both* types of consumers worse off. Low types pay $\frac{w}{2\bar{\alpha}} = \frac{w}{\alpha_\ell + \alpha_h}$ less in add-on pricing game than in the one-good model, but they get a good that is $\frac{w}{\alpha_\ell}$ less valuable to them. High types pay $\frac{w}{\alpha_h} - \frac{w}{\alpha_h + \alpha_\ell}$ more in the add-on pricing game for the same good.

3. One can get some intuition for why profits increase in the add-on pricing model by thinking about firm 1's best response when firm 2 sets $p_{2L} = c + 1/\alpha_\ell$ and $p_{2H} = c + 1/\alpha_h$. In the standard pricing model, firm 1's best response is to match these prices. In the add-on pricing model if $w/\alpha_h > 1/\alpha_h - 1/\alpha_\ell$ then it is as if firm 1 is constrained to choose prices that are farther apart. It thus chooses $p_{1L} < p_{2L}$ and $p_{1H} > p_{2H}$. Why do average prices increase? Roughly, prices are reduced less in the small market because cutting prices to the low types is more costly than increasing prices to the high types. Formally, the best-response prices satisfy the first order condition: $\frac{d\pi_{1L}}{dp_{1L}}(p_{1L}) = -\frac{d\pi_{1H}}{dp_{1H}}(p_{1H})$. Approximating the derivatives in a neighborhood of p_{2L} and p_{2H} using a second-order Taylor expansions gives

$$\frac{p_{2L} - p_{1L}}{p_{1H} - p_{2H}} = \frac{d^2\pi_{1H}/dp_{1H}^2}{d^2\pi_{1L}/dp_{1L}^2} = \frac{Q_H''(p_{1H})(p_{1H} - c) + 2Q_H'(p_{1H})}{Q_L''(p_{1L})(p_{1L} - c) + 2Q_L'(p_{1L})}$$

In the competition-on-a-line model, firm-level demand curves are linear, so the Q'' terms are zero and the fact that the low types' demand is more price-sensitive implies that p_{1L} is moved down from p_{2L} less than p_{1H} is moved up from p_{2H} . For more general demand curves, the calculation suggests that similar results may obtain when demand is convex or when it concave with $|Q''|$ not too large.

4. Good L can easily be sold at a loss in the add-on pricing model. Its price, $c + \frac{1}{\alpha} - \frac{w}{2\bar{\alpha}}$, is less than c whenever $w > 2$. The upper bound \bar{w} on w in the proposition is greater than 2 when $\alpha_\ell/\alpha_h > \frac{7+\sqrt{45}}{2} \approx 6.85$. The reason why good L is priced below cost (to attract consumers who will buy other goods at a higher markup) is the same as in Lal and Matutes. One difference in the outcome is that in my model, there are consumers who purchase only the loss leader. The benefit that the firms receive from having adopted the add-on pricing policy is also different. In Lal and Matutes not advertising one good is useful because the firms spend less on advertising. In my model, not advertising the price of good H softens price competition.

5. The multiplicity of equilibria in the standard pricing game is a consequence of firm 1's best responses in the two markets being more spread out when firm 2's prices are more spread out. When w is not too large the key constraint on the uniform pricing equilibrium is that firm 1 must prefer selling H to everyone to deviating and selling L to the low types and H to the high types. The key constraint on the discriminatory equilibrium is the exact opposite – firm 1 must prefer selling L to the low types and H to the high types to deviating and selling H to everyone. Selling both products at prices at least w apart is more attractive when the other firm is discriminating, and hence it is possible for the two equilibria to coexist.

6. Something that I have not discussed in detail in the proposition is that the add-on game will typically have a large number of equilibria with different payoff levels. The source of the multiplicity is that the Diamond result about the monopoly price being the unique outcome of a search model does not carry over to a model like the one given here with a discrete set of types. In addition to the Diamond-like continuation equilibrium where the firms both sell the upgrade at a price of w/α_h , there is also often a continuation equilibrium where consumers expect both firms to set a price of w/α_ℓ and firms set this price. Firms cannot profitably deviate from this equilibrium by raising the add-on price to $w/\alpha_\ell + \epsilon$ because this causes a discrete drop in demand. They cannot deviate by raising the add-on price to w/α_h because many of the high types would refuse to buy anything and instead go to the other firm where they expect that the upgrade price will be lower. This multiplicity of equilibria at $t = 2$ leads to additional equilibria in the whole game in a couple of ways. First, by assuming that the firms set $p_{iU} = w/\alpha_\ell$ whenever possible, one can sometimes (but not always) resurrect the equilibrium of proposition 1 in which $p_H^* = c + 1/\bar{\alpha}$. (Profits in the add-on pricing game are then lower than in the discriminatory equilibrium of the standard pricing game). It does not seem completely unreasonable to imagine that this equilibrium could prevail and that firms might sometimes miss out on an opportunity to benefit from add-on pricing because they are trapped in a lower profit equilibria by consumer expectations that add-ons will be reasonably priced. Second, one can construct bootstrapped equilibria with higher and lower profit levels by assuming that the firms set $p_{iU} = w/\alpha_h$ on the equilibrium path but punish each other by reverting to the equilibrium with $p_{iU} = w/\alpha_\ell$ following any deviation. These equilibria seem unreasonable.

5 Why do firms adopt add-on pricing?

The previous sections examine the impact of the joint adoption of add-on pricing. They do not address the question of why firms adopt add-on pricing. Profits are higher with add-on pricing because it is as if firms in the model were committed to keep p_{iH} and p_{iL} farther apart than they otherwise would be. A consequence of this is that if firms had the option of posting both prices at $t = 1$, they would want to deviate from the add-on pricing strategy and post both a higher price for good L and a lower price for good H at $t = 1$.

How can one account for the use of add-on pricing strategies? I list below a number of reasons why firms might endogenously choose to only post the price of good L in a model where firms have the option to post any number of prices at $t = 1$. Explanation 1 is a multiple equilibrium story in which add-on pricing is one of many equilibria; explanation 3 identifies a situation in which add-on pricing is the unique equilibrium, and the others are somewhere in between. In many cases, the explanations can be combined, e.g. one could argue that firms may choose not to post the price of good H because the potential gains are offset by a combination of incremental advertising costs and the desire to exploit boundedly rational consumers. Explanation 5 works only in conjunction with one or more of the others.

The previous sections can be seen as examining what happens when any of the root causes mentioned in this section leads firms to choose the timing of price postings that the add-on pricing model assumes.

1. Advertising costs determined by consumer search patterns

In many industries it would be prohibitively expensive to inform potential customers of a product's price via advertising. Hotels and car rental agencies, for example, serve consumers from all over the country and sell goods at many different prices. Avis would be crazy to conduct a nationwide media campaign to tell a few potential consumers that the rate for a three-day rental of a Pontiac Grand Am at the Detroit airport on August 2, 2002 is \$74.97. Instead, consumers learn about prices by actively looking for prices that firms have posted.

Firms can only cheaply inform consumers of prices that the consumers are looking for. If firms are known to use add-on pricing policies, consumers may only look for the prices of

low-quality goods. Each of the main internet travel websites, for example, is only designed to let consumers search for the base price for a rental, not for the price of a rental including insurance, prepaid gasoline, and other add-on charges.

If most consumers only look for prices for good L , add-on pricing will be individually rational. Cutting the price of good H lowers the firm's margin on all good H sales and does nothing to attract consumers who only look for good L prices.

2. Tacit collusion

The main conclusion of Section 4 was that the joint adoption of add-on pricing policies increases profits. This makes it attractive to tacitly collude on using an add-on strategy. To complete this story, one would want to explain why firms only collude on using add-on pricing rather than colluding on price. Colluding on price would be more profitable, so this presumably requires arguing that colluding on using add-on pricing is easier than colluding on price. Colluding on price can be difficult for many reasons: firms need to coordinate on changing prices in response to cost or demand shocks; firms may prefer different prices; and monitoring deviations from optimal pricing may be difficult if (as presumably happens with hotels, rental cars, etc.) the optimal pricing policy involves dynamically changing prices in response to privately known cost shocks and capacity constraints. A tacit agreement to use add-on pricing avoids all of the complexity, coordination, and monitoring issues: the firms just need to agree to and monitor that no one is advertising the price of good H .

To make this story more convincing, one would also want to argue not just that full collusion is impossible, but also that there aren't easy strategies for colluding on prices that are less than fully collusive but still are more profitable than the equilibrium prices in the add-on pricing game. See Athey, Bagwell and Sanchirico (2003) for a discussion of partially collusive pricing schemes in a model where firms have private information.

3. Exploitation of boundedly rational consumers

I mentioned earlier that the add-on pricing model can be given a "behavioral" interpretation: some or all of the high types could be unsophisticated consumers who are not as good at making price comparisons across firms and who are also easier to talk into buying add-ons at the point-of-sale. For example, they might be people who eat the jar of nuts sitting next to the minibar without realizing that it is part of the minibar and costs \$8

or people who think that they will pay their credit card bills on time and don't pay much attention to the late-payment fee when choosing a card even though they will likely end up making several payments late.

One reason why firms adopt add-on pricing policies may be that they somehow "trick" unsophisticated consumers into paying more than they would if the firms fully informed consumers about prices. For example, it seems plausible that some customers who are attracted by an advertisement for a \$99.95 weekly car rental and then agree to pay an extra \$91 over the course of week for insurance might have made other arrangements if the advertisement had listed both prices and made them think about how much the insurance would cost.

Add-on pricing can easily be individually rational for the firms if a fraction of the high types are unsophisticated consumers. The potential gain from selling to additional sophisticated high types at a lower price could be more than offset by losses that would result from not tricking the unsophisticated ones.

4. Per-product advertising costs

As in Lal and Matutes (1994), one could argue that it is sometimes more costly to advertise the prices of two products than the price of one product. If the incremental cost of advertising a second price is greater than the amount that a firm can gain by choosing a somewhat lower price for good H and a somewhat higher price for good L , then it will be individually rational for the firms to advertise just one price.

To make this a complete explanation for add-on pricing, one must also argue that firms cannot profitably deviate by posting a price for good H instead of a price for good L . If a firm only posts a price for good H at $t = 1$, then it will only sell good H in equilibrium. (The firm cannot set a price for good L that makes positive sales because the firm will always want to deviate and increase its good L price slightly given the search costs.) If firm 1 deviates from the add-on pricing equilibrium and sells good H to both populations, then its profits are bounded above by the profits it receives when it chooses the price p_{1H} to maximize

$$\pi_1(p_{1H}) = \left(\frac{1}{2} + \frac{\alpha_\ell}{2} \left(c + \frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} + \frac{w}{\alpha_\ell} - p_{1H} \right) \right) (p_{1H} - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2} \left(c + \frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} + \frac{w}{\alpha_h} - p_{1H} \right) \right) (p_{1H} - c).$$

This expression is maximized at $p_{1H} = c + 1/\bar{\alpha} + w/4\bar{\alpha}$ with the maximized value being

$(1 + w/4)^2/\bar{\alpha}$. For the parameter values considered in Proposition 2, this is less than the equilibrium profit, and hence sufficiently high per-product advertising costs will justify the add-on pricing equilibrium. The prices given in Proposition 2 are an equilibrium of the add-on pricing game for a larger set of parameter values than is covered by the hypotheses of the proposition. For some of these (e.g. when w is very large) the prices would fail to be an equilibrium of the game where firms can choose which price to advertise because the firms would want to deviate and advertise good H instead.

5. The difficulty of undercutting a nonposted price

Another factor that can be combined with any of the explanations above is that undercutting a high add-on price is more difficult than undercutting a high posted price or undercutting a high nonposted price in a single good model like Diamond's. The problem is that posting a lower price for good H may lead a firm's rival to also choose a lower add-on price (and consumers will anticipate this).

Consider, for example, the add-on pricing model with $\alpha_\ell/\alpha_h = 3$ and $w = 10/3$. The equilibrium has both firms set $p_L^* = c - 1/\alpha_\ell$ and $p_H^* = c + 3/\alpha_h$. If firm 2 was committed to these prices and firm 1 was capable of posting two prices at $t = 1$, its optimal deviation would be to dump all the unprofitable low types on the other firm and steal all of the high types by setting $p_{1L} \geq c$ and $p_{1H} = c + 2/\alpha_h$. In a one-good model, advertising a lower price and capturing the whole market is not difficult – we can construct such an equilibrium by assuming that consumers rationally believe that the firm which has been undercut continues to charge the monopoly price (and receives no visitors). This, however, is not possible in the add-on pricing model. If firm 1 makes the deviation described above, firm 2 would be visited by low types and only low types. Hence, it is not an equilibrium for firm 2 to continue to choose a high price for the add-on at $t = 2$. Instead the only equilibrium of the continuation game is for firm 2 to set $p_{2H} = p_{2L} + w/\alpha_\ell$. At this price, firm 2 sells to all of the low types and all of the high types, and firm 1 ends up with zero profits.

The extremely low profit resulting from this one poorly thought out deviation does not indicate that there is no profitable deviation from the profile above. Firm 1 simply needs to make a smaller deviation and be sure to leave its rival with enough high types so

that it remains an equilibrium for firm 2 to choose a high add-on price at $t = 2$. (In this case it must ensure that $q_{2H} \geq q_{2L}/2$.) This is not a problem for very small deviations, so firm 1 will always have a profitable deviation if there are no advertising costs. It is, however, an additional constraint that one must impose in computing the profits from the optimal deviation. In conjunction with per-product advertising costs or some other factor, the difficulty of undercutting a nonposted price may thus be thought of as another reason why posting only the good L price at $t = 1$ may be individually rational.

6 The cheapskate externality

How do cheapskates affect markets? The question may be current interest given that the internet makes it much easier for cheapskates to find and exploit small price differences. The standard answer would be that cheapskates play an important role in keeping prices near cost. Frankel (1998), for example, proposes that the desire to live where budget-conscious consumers keep prices low may be one reason why wealthy and poor households are often found in close proximity in the U.S. In this section, I note that the traditional view of cheapskates is turned on its head in the add-on pricing model.

The model of this section is a slight variant of the previous add-on pricing model that I will refer to as the “cheapskate model”. The only differences are that I assume that there is only an ϵ mass of cheapskates (rather than a unit mass) and that I will focus on what happens when α_ℓ is much larger than α_h .

Propositions 1 and 2 each contrast the outcome of the standard pricing game with the outcome of the add-on pricing game. Proposition 3 contrasts the outcome of the cheapskate version of the add-on pricing game with what would happen if firms were selling a single good to the same population. Part (a) illustrates that the ordinary intuition about the effects of cheapskates on other consumers and on firms is borne out in a one-good model, which can be obtained as a special case of the cheapskate model by assuming that $w = 0$. Part (b) notes that the ordinary comparative statics are reversed in the cheapskate model when w is large enough to act as a constraint forcing firms to keep prices for good L and H apart.⁸ One can thus think of add-on pricing as a practice that firms can adopt to turn

⁸As in Proposition 2 the requirement is that the upgrade price w/α_h be larger than what the difference

the presence of cheapskates from a curse into a blessing. At the same time the presence of cheapskates reduces the utility of normal consumers.

The intuition for the contrast is that whereas firms in the one-good model are tempted to slightly undercut each other to attract cheapskates, firms in the add-on pricing model are tempted to slightly overcut each other. When w is large, firms are losing money on the cheapskates and would like to dump all of their cheapskate customers on the other firm. When w is not quite so large, the firms earn positive profits on the cheapskates. However, if they were to leave the high price unchanged and sell L at $c + 1/\alpha_h - w/\alpha_h$, they would be selling L for less than $c + 1/\alpha_\ell$ and hence would prefer to serve fewer cheapskates at a higher margin.

Proposition 3 *Suppose $\alpha_\ell/\alpha_h > 2$. Define $\bar{\alpha}_\epsilon \equiv \frac{\alpha_h + \epsilon\alpha_\ell}{1 + \epsilon}$.*

(a) *In the one-good version of the cheapskate model obtained by setting $w = 0$, for sufficiently small ϵ the unique symmetric equilibrium has $p^* = c + 1/\bar{\alpha}_\epsilon$, and prices and profits are decreasing in ϵ .*

(b) *If $w > \underline{w}$ (as defined in Proposition 2), then for sufficiently small ϵ the unique symmetric equilibrium of the cheapskate version of the add-on pricing model has*

$$p_H^* = c + \frac{1}{\alpha_h} + \left(\frac{w}{\alpha_h} - \left(\frac{1}{\alpha_h} - \frac{1}{\alpha_\ell} \right) \right) \frac{\epsilon\alpha_\ell}{\alpha_h + \epsilon\alpha_\ell},$$

and profits and the price paid by high types are increasing in ϵ .

Proof

(a) In a neighborhood of any symmetric equilibrium price p^* firm 1's profits when $\epsilon = 0$ are

$$\pi_1(p_1) = \left(\frac{1 + \epsilon}{2} + \frac{\alpha_h + \epsilon\alpha_\ell}{2} (p^* - p_1) \right) (p_1 - c).$$

The first order condition for maximizing this implies that the only possible symmetric pure strategy equilibrium is $p^* = c + 1/\bar{\alpha}_\epsilon$. To verify that this is indeed an equilibrium one must also check that firm 1 cannot profitably deviate to a higher price at which it serves no low types. The price that maximizes firm 1's profits from sales to high types is $p_1 = c + \frac{1}{2\bar{\alpha}_\epsilon} + \frac{1}{2\alpha_h}$. The profits from the high types at this price are $\frac{\alpha_h}{8} \left(\frac{1}{\alpha_h} + \frac{1}{\bar{\alpha}_\epsilon} \right)^2$. One can show that this is less than the equilibrium profit level for sufficiently small ϵ by evaluating the derivatives between p_H and p_L would be if the firms competed separately for the low and high types.

of this expression and the expression for the equilibrium profits with respect to ϵ at $\epsilon = 0$. Intuitively, if the firm abandons the low market it gives up a potential profit that is first-order in ϵ , while the profits that a firm sacrifices in the high market when it also serves the low types are second-order in ϵ by the envelope theorem (because the price is approaching the optimal price in the high submarket).

The expression for the equilibrium price is clearly decreasing in ϵ . Equilibrium profits are given by $\frac{(1+\epsilon)^2}{\alpha_h + \epsilon\alpha_\ell}$. Evaluating the derivative of this expression with respect to ϵ at $\epsilon = 0$ it is immediately evident that profits are decreasing in ϵ a neighborhood of $\epsilon = 0$ if $\alpha_\ell > 2\alpha_h$.

(b) Let p_L^* be the price set at $t = 1$ in a pure strategy equilibrium. When ϵ is sufficiently small both firms will set $p_{iH} = p_{iL} + w/\alpha_h$ at $t = 2$ whenever the first period prices are in some neighborhood of p_L^* . Hence, if firm 1 deviates to a price in a neighborhood of p_L^* its profits are given by

$$\pi_1(p_{1L}) = \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_L^* - p_{1L}) \right) (p_{1L} + w/\alpha_h - c) + \epsilon \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_L^* - p_{1L}) \right) (p_{1L} - c).$$

The fact that any equilibrium price p_L^* must be a solution to the first order condition for maximizing this expression gives that the only possible equilibrium is to have p_L^* equal to w/α_h less than the expression given in the statement of the proposition. The expression for p_H^* is clearly increasing in ϵ . A first-order approximation to the profits when the firms charge prices p_L^* and p_H^* is

$$\pi^*(\epsilon) = \frac{1}{2\alpha_h} + \frac{1}{2} \left(\frac{\alpha_\ell}{\alpha_h} \left(\frac{w}{\alpha_h} - \left(\frac{1}{\alpha_h} - \frac{1}{\alpha_\ell} \right) \right) + \frac{1-w}{\alpha_h} \right) \epsilon + O(\epsilon^2).$$

The coefficient on ϵ in this expression is positive when $w = \alpha_h(1/\alpha_h - 1/\alpha_\ell)$, and the coefficient is increasing in w . Hence, for all w satisfying the hypothesis of part (b), profits are increasing in ϵ when ϵ is small.

To complete the proof of part (b), it remains only to show that the prices derived above are an equilibrium and not just the solution to the first-order condition. Deviating to a higher price cannot be profitable. The concave profit function above applies as long as sales to the low types are nonzero. Hence firm 1's profits decline as it raises its price from p_L^* to $p_L^* + 1/\alpha_\ell$. Any price increases beyond that point would further decrease profits as profits, since profits from sales to the high types are decreasing in p_{1L} at p_L^* and all higher prices. No deviation to a lower price will be profitable if firm 1 makes positive sales to the low

types at the price which maximizes its profits on sales to the high types (by the concavity of the profit function). The difference between p_H^* and the price that maximizes profits from sales to the high types (setting $p_{1H} = \frac{1}{2}(p_H^* + c + 1/\alpha_h)$) is of order ϵ . Hence for ϵ small it is within $1/\alpha_\ell$ of the equilibrium price and we can conclude that the profile is an equilibrium.

QED

7 Related literature

I see Holton (1957) as the seminal paper on loss-leaders in multigood settings. It notes that “The margin sacrificed on the loss leader is, of course, a promotion expense incurred to boost the sales of the other products of the store” and argues that high margins on the “other” products can be rationalized because “the supermarket enjoys a spatial monopoly on that item *once the consumer is in the store.*”⁹ Holton, of course, was writing before the advent of modern oligopoly theory and could not address the impact of loss-leader tactics on equilibrium prices and profits. The modern IO literature has also not identified the effect of loss leader pricing on equilibrium profits highlighted in this paper.

Most closely related are two papers that have used similar models to explain why loss leaders can be sold at a loss and why high quality products are sold at higher markups. The first of these, Lal and Matutes (1994), is the starting point for this paper – two firms each sell two goods to a continuum of consumers located along a Hotelling line. Firms advertise one product as a loss leader and recoup the losses by holding consumers up for their reservation value on the unadvertised product. Lal and Matutes do emphasize that loss-leader pricing increases profits, but this occurs only because loss leaders allow firms to economize on per-product advertising costs. Apart from advertising cost savings, their model is one in which add-on pricing is irrelevant.

The second, Verboven (1999), focuses on why premium quality products have higher markups. The reason he proposes is exactly the same as in Lal and Matutes – he assumes that there is a Diamond-like search game in the second stage that results in firms charging the monopoly price for the add-on. His paper differs from Lal and Matutes in the functional

⁹The emphasis is in the original. See pages 21 and 27 of Holton (1957).

forms on preferences: the horizontal taste preferences are logit rather than uniform; and he also assumes that consumers are vertically differentiated with the marginal value of quality being uniformly distributed. The vertical differentiation means that some consumers will benefit from add-on pricing and some will be hurt. The firms, however, remain exactly indifferent. A crucial difference between Verboven's model and the model of this paper is that Verboven's vertical heterogeneity is in the marginal valuation of quality, rather than the marginal utility of income. Hence, price cuts attract low- and high- types equally, rather than disproportionately attracting customers who will refuse to buy the add-on.

Simester (1995) and Lazear (1995) provide less related models of similar tactics. Simester (1995) provides a signalling explanation for loss leaders. The model has two retailers located at the opposite ends of a Hotelling line selling two products each. Firms have private information about their per unit retailing costs and can advertise only one product. Firms with lower costs set lower prices for the unadvertised goods. Prices for advertised goods may be distorted downward to signal that a firm's unadvertised goods have low prices. Lazear (1995) is a monopoly model of bait-and-switch advertising. The firm has available for sale only one of the two potential products A and B . Customers have heterogeneous preferences for A and B and incur a transportation cost in visiting the firm. One might think that firms would always advertise their price for the product they have in stock. Lazear notes, however, that sometimes there is instead or in addition a pooling equilibrium where all firms advertise B regardless of what they have in stock. This occurs when an advertisement for B would attract a large number of consumers who are unwilling to pay the transportation cost to get the opportunity to buy A , but who may be willing to buy A once they are at the store and the cost is sunk.

Hess and Gerstner (1987) examines a motivation to stock out on advertised products and offer rain checks. The model involves Bertrand competition between retailers selling two products: a primary product which consumers purchase once from the store that has the best combination of a low price and a high likelihood of having the product in stock; and an "impulse good" that consumers purchase every week without shopping around. Firms offer some rain checks in equilibrium because issuing a rain check guarantees that the consumer will return the following week and make his impulse purchase from the same store. With Bertrand competition, the profits from the subsequent visit are returned to consumers in

lower prices for the primary good. In equilibrium all firms adopt the rain-check strategy and profits are lower than if rain checks were banned.

The literature on competitive price discrimination has asked some related questions. Holmes (1989) examines banning price discrimination when duopolists compete in two separate markets. While monopolists are always hurt by restrictions on prices, duopolists need not be. Banning price discrimination lowers prices in one market and raises them in the other; the net effect on profits is ambiguous. Corts (1997) examines two vertically differentiated firms selling one product each. There are two groups of consumers: one values quality and one does not (and hence considers products undifferentiated). He finds that banning price discrimination may increase prices in both markets. One could think of this conclusion as the exact opposite of remark 2 in Section 4, but the models and mechanisms are mostly unrelated so this is probably not enlightening. Lal and Matutes (1989) examines a model more similar to that of this paper (without unadvertised prices), but the ideas it develops are not closely related. It shows that despite the perfect information, firms may achieve the fully collusive profit in an asymmetric equilibrium.

This paper can also be thought of as related to any paper discussing a strategic investment that softens competition. Chapter 8 of Tirole (1988) reviews a number of such papers. A classic example is Thisse and Vives (1988), which notes that firms are better off competing in FOB prices than in delivered prices, because when they choose separate delivered prices for each location they end up being in Bertrand competition for the consumers at each location. As in this paper, they also note that FOB pricing is not individually rational in an extended game in which firms first choose pricing policies, and then compete in prices.

This paper is contributing to these literatures in a few ways: it analyzes a structure of consumer preferences that seems more realistic for some applications; it reaches a different conclusion on the price and welfare effects of add-on pricing strategies; its contribution to the price discrimination literature is to examine a competitive model of second-degree discrimination; and it notes that the constraint that firms must hold consumers up for the full value of unadvertised add-ons softens competition.

The one very closely related empirical paper is Ellison and Ellison (2002), which analyzes demand and markups at a retailer using an add-on strategy when selling computer parts on the internet. It provides evidence in support of this paper in two ways: it provides

evidence that this paper's assumptions about demand reflect reality in at least one market; and it provides evidence in support of this paper's conclusions. The evidence relevant to the assumptions are estimates of how the demand for products of several quality levels depends on the prices of all of the other qualities. Specifically, loss leaders are shown to attract a large number of customers who end up buying upgraded products at higher price, and there is evidence of the adverse selection effect – the customer pool of attracted by a low-priced loss leader is shown to have a much higher percentage of customers who do not upgrade. Supporting evidence for the conclusion that add-on pricing softens competition comes from a straightforward analysis of price and cost data. The firm is estimated to earn average markups over marginal cost of 10 to 15 percent even though the elasticity of demand for the base good is about -40.

There is surprisingly little other empirical evidence on loss-leader pricing. The one standard empirical reference in marketing seems to be Walters (1988). It examines the impact of loss leaders on store traffic by estimating a system of simultaneous equations. The key equation essentially regresses the total number of customers visiting a supermarket in a week on dummy variables for whether a product in each of eight categories is featured in a sales circular and offered at a discount of at least 15%. Walters finds little evidence that loss leaders affect store traffic. Chevalier, Rossi, and Scharfstein (2003) use data from a Chicago supermarket chain to examine the pricing and demand for products that have large seasonal peaks in demand. Several findings are consistent with these products serving as loss leaders: the retail margin of a product tends to decline during the period of its peak demand even if this does not coincide with a peak in aggregate supermarket demand; aggregate margins do not decrease during aggregate demand peaks; reductions in item prices during product-specific demand peaks do not appear to be due to changes in demand elasticities; and reductions in item prices during product-specific demand peaks are associated with increases in product-specific advertising. Verboven (1999) uses a hedonic regression to compare markups for base model cars and cars with more powerful engines and finds that percentage markups on the premium engines are higher in some car classes but not in others.

8 Conclusion

The add-on pricing strategy described in this paper could be practiced in almost any business. Firms just need to be able to invent a lower-quality versions of their products; the lower-quality products need not be any cheaper to produce. The key feature of the consumer pool is that consumers who are more sensitive to inter-firm price differences are less likely to purchase costly add-ons. This seems plausible given a number of sources of heterogeneity, e.g. rich versus poor consumers, individual versus business customers, or sophisticated versus unsophisticated shoppers.

The general idea of creating intentionally creating an adverse selection problem to limit competition is perhaps also one that could be applied in contexts other than pricing games.

For firms the main consequence of add-on pricing is that profits are higher than they otherwise would be given the degree of product differentiation. This effect may be generally important to our understanding of how firms maintain sufficient markups to survive in a world where fixed costs are often substantial. In the long run, of course, entry would be expected to reduce the degree of differentiation between adjacent firms and bring profits into line with fixed costs. What add-on pricing may help us understand is thus why we observe so many firms in various industries.

I have not discussed social welfare. Models with unit demands are poorly suited to welfare analyses. For example, social welfare in the add-on pricing model is identical to that in the discriminatory equilibrium of the standard pricing model – in both models all low types buy one unit of L and all high types buy one unit of H . In a more realistic setup, the lower price for good L would increase consumption of L and the higher price for the add-on would reduce consumption of H . How the losses and gains would trade off is not clear. The welfare comparison between the add-on pricing model and the one-good model obtained by eliminating good L may be more straightforward. I noted that both the high and low types pay more relative to their valuation in the add-on pricing game than in the one-good model. If this is also true in a model with continuous aggregate demand functions, deadweight loss would presumably be unambiguously larger in the add-on model. (Welfare is unambiguously lower in the add-on pricing game with unit demands because it is inefficient for the low types to buy L rather than H .)

References

- Athey, Susan, Kyle Bagwell and Chris Sanchirico (2003): "Collusion and Price Rigidity," *Review of Economic Studies*, forthcoming.
- Chevalier, Judith, Anil Kashyap and Peter Rossi (2003): "Why Don't Prices Rise During Periods of Peak Demand? Evidence from Scanner Data," *American Economic Review* 93, 15-37.
- Corts, Kenneth (1998): Third-Degree Price Discrimination in Oligopoly: All-out Competition and Strategic Commitment," *Rand Journal of Economics* 29, 306-323.
- Deneckere, Raymond J. and R. Preston McAfee (1996): "Damaged Goods," *Journal of Economics and Management Strategy* 5, 149-174.
- Diamond, Peter (1971): "A Model of Price Adjustment," *Journal of Economic Theory* 3, 156-168.
- Ellison, Glenn and Sara F. Ellison (2002): "Search, Obfuscation, and Price Elasticities on the Internet," MIT, mimeo.
- Frankel, David (1998): "A Pecuniary Reason for Income Smoothing," *Journal of Urban Economics* 44, 158-169.
- Hess, James D. and Eitan Gerstner (1987): "Loss Leader Pricing and Rain Check Policy," *Marketing Science* 6, 358-374.
- Holmes, Thomas J. (1989): "The Effects of Third-Degree Price Discrimination in Oligopoly," *The American Economic Review* 79, 244-250.
- Holton, Richard H. (1957): "Price Discrimination at Retail: The Supermarket Case," *Journal of Industrial Economics* 6, 13-32.
- Lal, Rajiv and Carmen Matutes (1989): "Price Competition in Multimarket Duopolies," *Rand Journal of Economics* 20, 516-537.
- Lal, Rajiv and Carmen Matutes (1994): "Retail Pricing and Advertising Strategies," *Journal of Business* 67, 345-370.
- Lazear, Edward P. (1995): "Bait and Switch," *Journal of Political Economy* 103, 813-830.
- Pratt, John, David A. Wise and Richard J. Zeckhauser (1979): "Price Differences in Almost Competitive Markets," *Quarterly Journal of Economics* 93, 189-211.
- Simester, Duncan (1995): "Signalling Price Image Using Advertised Prices," *Marketing Science* 14, 166-188.
- Thisse, Jacques-Francois, and Xavier Vives (1988): "On the Strategic Choice of Spatial Pricing Policy," *American Economic Review* 78, 122-137.
- Tirole, Jean (1988): *The Theory of Industrial Organization*. Cambridge MA: MIT Press.

Verboven, Frank (1999): "Product Line Rivalry and Market Segmentation—With an Application to Automobile Optional Engine Pricing," *Journal of Industrial Economics* 47, 399-425.

Walters, Rochney (1988): "Retail Promotions and Retail Store Performance: A Test of Some Key Hypotheses," *Journal of Retailing* 64, 153-180.

Appendix

Proof of Proposition 1

(a) Consider first the possibility of a symmetric pure strategy Nash equilibrium where all consumers buy good H at a price of p_H^* . This requires that $p_{iL} \geq p_H^* - w/\alpha_\ell$. If firm 1 deviates to a price p_{1H} in a neighborhood of p_H^* (and raises p_{1L} at the same time if need be) then firm 1's profits are

$$\pi_1(p_{1H}) = \left(1 + \frac{\alpha_\ell + \alpha_h}{2}(p_H^* - p_{1H})\right)(p_{1H} - c)$$

A necessary condition for Nash equilibrium is that the derivative of this expression be zero at $p_{1H} = p_H^*$. This gives $p_H^* = \frac{1}{2}\left(c + \frac{1}{\bar{\alpha}} + p_H^*\right)$, which implies that the only possible equilibrium of this form is $p_{1H} = p_{2H} = p_H^* = c + 1/\bar{\alpha}$.

To show that it is indeed a SPE for both firms to set $p_{iH} = c + 1/\bar{\alpha}$ and $p_{iL} \geq c + 1/\bar{\alpha} - w/\alpha_\ell$ (with all consumers buying good H from the closest firm) requires that we check that various possible deviations do not increase a firm's profits.

Consider first a deviation to prices p_{1L} and p_{1H} at which consumers only buy good H . To show that such a deviation cannot increase firm 1's profits I'll make a few observations in succession.

Observation 1: If firm 1 sells good H to some but not all consumers in each population then the deviation does not increase profits.

To see this, note that in this case the formula above gives firm 1's profits. The expression is a quadratic in p_{1H} and hence the solution to the first-order condition is the maximum.

Observation 2: If firm 1 sells good H to everyone in the cheapskate population then the deviation does not increase profits.

With such prices, firm 1's profits are smaller than what one gets from plugging p_{1H} into the profit formula above, which in turn is smaller than the profits from setting $p_{1H} = p_H^*$.

Observation 3: If firm 1 makes no sales in the cheapskate population then the deviation is not profitable.

If firm 1 chooses $p_{1H} > p_H^* + 1/\alpha_\ell$ then it makes sales only to the high types and its profits are

$$\pi_1(p_{1H}) = \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_H^* - p_{1H})\right)(p_{1H} - c)$$

Taking the first order condition we see that the global maximum of this expression occurs at

$$p_{1H} = c + \frac{1}{2\alpha_h} + \frac{1}{2\bar{\alpha}}.$$

The firm would sell to low types at this price if

$$c + \frac{1}{2\alpha_h} + \frac{1}{2\bar{\alpha}} \leq c + \frac{1}{\alpha} + \frac{1}{\alpha_\ell}.$$

A straightforward calculation shows that this is the case if $\alpha_\ell/\alpha_h \leq (3 + \sqrt{17})/2 \approx 3.562$, which is true given the assumption of the Proposition. Hence, we can conclude that the profits from any price that sells only to the high types are at most equal to the profits received from the high types by setting $p_{1H} = c + \frac{1}{2\alpha_h} + \frac{1}{2\bar{\alpha}}$, which in turn is less than the profits received from setting this price and selling to members of both populations, which by observation 1 are less than what firm 1 receives by setting $p_{1H} = p_H^*$.

Taken together, observations 1-3 imply that any deviation which involves only selling good H is not profitable: if firm 1 deviates to $p_{1H} < p_H^*$ then firm 1 makes more sales to cheapskates than to high types so either observation 1 or observation 2 applies; if firm 1 deviates to $p_{1H} > p_H^*$ then firm 1 makes more sales to high types than to cheapskates and observation 1 or observation 3 applies.

Observation 4: Any deviation to prices p_{1L} and p_{1H} at which firm 1 sells only good L is not profitable.

To see this, note that firm 1 would sell at least as many units (and get a higher price on each at no higher cost) by setting prices $p'_{1L} = \infty$ and $p'_{1H} = p_{1L} + w/\alpha_\ell$. We've already shown these prices do not increase firm 1's profit.

Finally, consider a deviation to prices p_{1L} and p_{1H} at which firm 1 sells good L to the cheapskates and good H to the high types. If there were no IC constraints so firm 1 could simply choose the optimal prices in each population its choices would be $p_{1H} = c + \frac{1}{2\bar{\alpha}} + \frac{1}{2\alpha_h}$ and $p_{1L} = c + \frac{1}{2\bar{\alpha}} + \frac{1-w}{2\alpha_\ell}$. If $w < \frac{\alpha_\ell - \alpha_h}{2\alpha_\ell - \alpha_h}$, however, these prices would lead the high types to buy good L . If $w > \frac{\alpha_\ell - \alpha_h}{\alpha_h}$, these prices would lead the low types to buy good H . Accordingly, I will consider separately the optimal deviation of this form when w is small (with the high type's IC constraint binds), intermediate, and high (with the low type's IC constraint binding). I do this by presenting an additional series of observations.

Observation 5: If $w \leq \frac{\alpha_\ell - \alpha_h}{2\alpha_\ell - \alpha_h}$ then a deviation that sells L to the low types and H to the high types is not profitable.

In this case the constraint that $p_{1H} - p_{1L} \leq w/\alpha_h$ binds. Define $\pi_1(p_{1H}, w)$ by

$$\pi_1(p_{1H}, w) \equiv \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_H^* - p_{1H}) \right) (p_{1H} - c) + \left(\frac{1}{2} + \frac{\alpha_\ell}{2} \left(p_H^* - \frac{w}{\alpha_\ell} - \left(p_{1H} - \frac{w}{\alpha_h} \right) \right) \right) \left(p_{1H} - \frac{w}{\alpha_h} - c \right).$$

Let $\pi_1^d(w) = \max_{p_{1H}} \pi_1(p_{1H}, w)$ and write p_{1H}^* for the price that maximizes this expression. The maximum profit achievable by a deviation of this form is at most $\pi_1^d(w)$ as long as the best possible deviation of this form has $p_{1H} - w/\alpha_h \geq c$. (In the opposite case the deviation can't increase profits because firm 1 would be better off not selling good L and we have already seen that such deviations do not increase firm 1's profits.) From the envelope theorem we have have

$$\frac{d\pi_1^d}{dw} = \frac{\partial \pi_1}{\partial w} = \frac{1}{2\alpha_h} \left((2\alpha_\ell - \alpha_h)(p_{1H}^*(w) - c) - \frac{2w(\alpha_\ell - \alpha_h)}{\alpha_h} - \frac{\alpha_\ell}{\bar{\alpha}} - 1 \right).$$

To show that $\pi_1^d(w) < 1/\bar{\alpha}$ for all $w \in \left(0, \frac{\alpha_\ell - \alpha_h}{2\alpha_\ell - \alpha_h}\right)$ it suffices to show that the derivative is negative for all w in the interval. For this it suffices to show that

$$(2\alpha_\ell - \alpha_h)(p_{1H}^*(w) - c) < 1 + \alpha_\ell/\bar{\alpha}.$$

If the high type's IC constraint were not binding firm 1 would choose $p_{1H} = c + \frac{1}{2\bar{\alpha}} + \frac{1}{2\alpha_h}$. Given the constraint the optimal $p_{1H}^*(w)$ will be smaller. Plugging this upper bound into the equation above gives that a deviation is not profitable if

$$\frac{1}{2}(2\alpha_\ell - \alpha_h) \left(\frac{\alpha_h + \bar{\alpha}}{\alpha_h \bar{\alpha}} \right) < \frac{\bar{\alpha} + \alpha_\ell}{\bar{\alpha}}.$$

Multiplying through and collecting terms this is equivalent to

$$2\alpha_\ell^2 - \alpha_\ell \alpha_h - 5\alpha_h^2 < 0,$$

which holds provided that $\alpha_\ell/\alpha_h < (1 + \sqrt{41})/4 \approx 1.851$.

Observation 6: If $\frac{\alpha_\ell - \alpha_h}{2\alpha_\ell - \alpha_h} \leq w \leq \frac{\alpha_\ell - \alpha_h}{\alpha_h}$ then a deviation that sells L to the low types and H to the high types is not profitable.

In this case, the IC constraints are not binding and the optimal deviation of this form is to $p_{1L} = c + \frac{1}{2\bar{\alpha}} + \frac{1-w}{2\alpha_\ell}$ and $p_{1H} = c + \frac{1}{2\bar{\alpha}} + \frac{1}{2\alpha_h}$. With these prices profits from high type consumers are independent of w and profits from low type consumers are decreasing in w .

To see that the deviation is not profitable for any w in the interval it therefore suffices to show that the deviation is not profitable when $w = \frac{\alpha_\ell - \alpha_h}{2\alpha_\ell - \alpha_h}$. This follows from observation 5.

Observation 7: If $\frac{\alpha_\ell - \alpha_h}{\alpha_h} \leq w$ then a deviation that sells L to the low types and H to the high types is not profitable.

In this case, the IC constraint of the low type is binding. The optimal deviation of this type has $p_{1L} = p_{1H} - w/\alpha_\ell$. This can not increase firm 1's profits, because the type L consumers would also be willing to buy good H at price p_{1H} . Hence, firm 1 could do better selling only good H and we have already seen that there is no profitable deviation of this form.

This concludes the argument to show that there are subgame perfect equilibria with $p_{2H} = p_{2L} = c + 1/\bar{\alpha}$, $p_{iL} > c + 1/\bar{\alpha} - w/\alpha_\ell$ and all consumers buying H from the closest firm at $t = 3$.

To prove the uniqueness claim of part (a), we must also show that there are no other symmetric pure strategy equilibria in the standard pricing game. It is obvious that there are no equilibria in which all consumers buy good L . A firm could increase its profits by setting $p'_{1L} = \infty$ and $p'_{iH} = \min(c, p_{iL} + w/\alpha_\ell)$. There are no equilibria where the low types buy good H and high types buy good L because the high types will strictly prefer to buy H whenever the low types weakly prefer H .

The final more serious possibility to consider is whether there is an equilibrium in which low types buy good L and high types buy good H . We can think of three possible cases: equilibria where low types and high types both strictly prefer to purchase the good they are purchasing, those where the high types are indifferent to buying good L , and those where the low types are indifferent to buying good H . The last of the three cases is not possible — each firm could increase its profits by not offering good L (because its low type consumers would buy H instead at the higher price). I will first discuss the first case.

In a discriminatory equilibrium where low types strictly prefer good L and high types strictly prefer good H the first order conditions for each firm's profits imply that the only possible equilibrium is $p_{1L} = p_{2L} = c + 1/\alpha_\ell$ and $p_{1H} = p_{2H} = c + 1/\alpha_h$. Low types prefer good L at these prices only if $p_{iL} < p_{iH} - w/\alpha_\ell$. This requires $w \leq \frac{\alpha_\ell - \alpha_h}{\alpha_h}$. High types prefer good H at these prices only if $p_{iL} > p_{iH} - w/\alpha_h$. This requires $w \geq \frac{\alpha_\ell - \alpha_h}{\alpha_\ell}$. Assume that w

does satisfy these conditions.

Suppose that firm 1 deviates to $p'_{1L} = \infty$ and $p'_{1H} = c + \frac{1}{\bar{\alpha}} + \frac{w}{4\bar{\alpha}}$. One can verify that $p'_{1H} > p_{2H} - 1/\alpha_h$ and $p'_{1H} > p_{2L} + w/\alpha_\ell - 1/\alpha_\ell$ whenever $\alpha_\ell/\alpha_h < (3 + \sqrt{17})/2$. Hence, after the deviation firm 1 sells to a subset of each population and firm 1's profits are bounded below by the standard expression for profits in a competition-on-a-line model. Omitting much algebra this gives that the profits from the deviation are at least

$$\left(\frac{1}{2} + \frac{\alpha_h}{2}(p_{2H} - p'_{1H})\right)(p'_{1H} - c) + \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_{2L} - (p'_{1H} - w/\alpha_\ell))\right)(p'_{1H} - c) = \left(1 + \frac{w}{4}\right)^2 \frac{1}{\bar{\alpha}}$$

This is a profitable deviation from the hypothesized equilibrium profile if

$$\left(1 + \frac{w}{4}\right)^2 \frac{1}{\bar{\alpha}} > \frac{1}{2\alpha_\ell} + \frac{1}{2\alpha_h}.$$

Using the fact that $w/geq(\alpha_\ell - \alpha_h)/\alpha_\ell$ this shows that there is no equilibrium of this form if

$$\left(1 + \frac{\alpha_\ell - \alpha_h}{4\alpha_\ell}\right)^2 \frac{1}{\bar{\alpha}} > \frac{1}{2\alpha_\ell} + \frac{1}{2\alpha_h}.$$

Expanding the formula above we can see that this is true if and only if

$$\left(\frac{\alpha_\ell}{\alpha_h} - 1\right) \left(4\left(\frac{\alpha_\ell}{\alpha_h}\right)^2 - 13\frac{\alpha_\ell}{\alpha_h} + 1\right) < 0.$$

This is true for

$$1 < \frac{\alpha_\ell}{\alpha_h} < \frac{13 + \sqrt{153}}{8} \approx 3.171$$

The final analysis necessary to complete the proof of part (a) is a demonstration that there are also no discriminatory equilibria with $p_{iL} = p_{iH} - w/\alpha_h$ with the parameter restrictions of part (a). Firm 1 could deviate from such an equilibrium by raising or lowering p_{1L} and changing p_{1H} by exactly the same amount (i.e., setting $p_{1H} = p_{1L} + w/\alpha_h$). For a small enough change in prices firm 1 would continue to sell L to a fraction of the low types and H to a fraction of the high types. Firm 1's profit would then be

$$\pi_1(p_{1L}) = \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_{2L} - p_{1L})\right)(p_{1L} - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_{2L} - p_{1L})\right)(p_{1L} + w/\alpha_h - c).$$

Considering the first order condition for maximizing this expression we can see that the only possible SPE of this form would have $p_{1L} = c + 1/\bar{\alpha} - w/2\bar{\alpha}$ (and $p_{1H} = c + 1/\bar{\alpha} -$

$w/2\bar{\alpha} + w/\alpha_h$.) Given the restriction on α_ℓ/α_h in the proposition it turns out that there is always a profitable deviation from this profile.

If $w > (\alpha_\ell - \alpha_h)/\alpha_\ell$ a profitable deviation is to raise p_{1L} by a small amount and leave p_{1H} unchanged. With such a deviation profits from sales to the high types will be unchanged and firm 1 will sell fewer units of good L to low types (at a higher price). This is profitable if the derivative with respect to p_{1L} of

$$\left(\frac{1}{2} - \frac{\alpha_\ell}{2}(p_{1L} - p_{2L})\right)(p_{1L} - c)$$

is positive when evaluated at $p_{1L} = p_{2L} = c + 1/\bar{\alpha} - w/2\bar{\alpha}$. The derivative is

$$\frac{1}{2} - \frac{\alpha_\ell}{2} \left(\frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}}\right),$$

which is positive for $w > (\alpha_\ell - \alpha_h)/\alpha_\ell$.

When $w \leq (\alpha_\ell - \alpha_h)/\alpha_\ell$ a profitable deviation is to simply raise p_{1L} sufficiently high so that the low types will prefer to buy good H . Firm 1 will sell fewer units with this strategy, but at a higher price. Profits from the high types are unchanged. Profits from sales to the low types change from $\frac{1}{2}(1/\bar{\alpha} - w/2\bar{\alpha})$ to

$$\left(\frac{1}{2} - \frac{\alpha_\ell}{2} \left(\frac{w}{\alpha_h} - \frac{w}{\alpha_\ell}\right)\right) \left(\frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} + \frac{w}{\alpha_h}\right).$$

The change in profits simplifies to

$$\frac{w}{2} \left(\frac{1}{\alpha_h} - \frac{\alpha_\ell - \alpha_h}{\alpha_h} \frac{2\alpha_h + w\alpha_\ell}{\alpha_h(\alpha_\ell + \alpha_h)}\right).$$

Substituting in the upper bound $(\alpha_\ell - \alpha_h)/\alpha_\ell$ for the second w in this expression and simplifying we find that the change in profits is at least

$$\frac{w}{2} \frac{2\alpha_h - \alpha_\ell}{\alpha_h^2},$$

which is positive for $\alpha_\ell/\alpha_h < 2$. This completes the proof that there is no equilibrium in which the firms make sales of good L and thereby completes the proof of part (a) of the proposition.

(b) To analyze the add-on pricing game, I begin with a lemma noting that if the firms' first period prices are close together, then at $t = 2$ the firms will sell the "upgrade" to all consumers at a price of w/α_ℓ .

Lemma 1 Assume $\alpha_\ell/\alpha_h \leq 1.6$. Suppose that at $t = 1$ the firms choose prices p_{1L} and p_{2L} with $|p_{2L} - p_{1L}| \leq \frac{2\alpha_h - \alpha_\ell}{\alpha_h^2}$ and $c < p_{iL} < (v - w - s - 1/2)/\alpha_\ell$. Then, the unique equilibrium of the subgame at $t = 2$ has the firms selling the upgrade to all consumers at a price of w/α_ℓ .

A proof of the lemma is presented immediately after the proof of this Proposition. Given the result of the lemma, we know that firm 1's profit following a small deviation at $t = 1$ from the symmetric profile $p_{1L} = p_{2L} = p_L^*$ results in its earning a profit of

$$\pi_1(p_{1L}) = \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_{2L} - p_{1L}) \right) (p_{1L} + w/\alpha_\ell - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_{2L} - p_{1L}) \right) (p_{1L} + w/\alpha_\ell - c).$$

Considering the first order condition for maximizing this expression shows that the only possible first period price in a symmetric SPE is $p_L^* = c + 1/\bar{\alpha} - w/\alpha_\ell$. By Lemma 1, at $t=2$ both firms must set $p_{iH} = c + 1/\bar{\alpha} - w/\alpha_\ell + w/\alpha_\ell = c + 1/\bar{\alpha}$ on the equilibrium path, and all consumers must buy good H from the nearest firm. This completes the proof of the uniqueness part of part (b) of the proposition.

To verify that there is indeed a pure strategy SPE of the form described, suppose that both firms set $p_{iL} = c + 1/\bar{\alpha} - w/\alpha_\ell$ at $t = 1$ and follow some SPE strategy at $t = 2$ and that consumers behave optimally given the firms' equilibrium strategies and purchase good H if they are indifferent between buying H and L .

By definition we know that firm 1 has no profitable deviation at $t = 2$.

To show that there is no profitable deviation at $t = 1$, I will present a series of observations covering various cases.

Observation 1: Firm 1 cannot increase its profits by deviating to any p_{1L} with $|p_{1L} - p_L^*| < \frac{2\alpha_h - \alpha_\ell}{\alpha_h^2}$.

With such a deviation, Lemma 1 implies that firm 2 sets $p_{2H} = c + 1/\bar{\alpha}$ at $t = 2$. Part (a) of the proposition implies that no matter what prices p_{1L} and p_{1H} firm 1 chooses it cannot earn a profit in excess of $1/\bar{\alpha}$ when $p_{2H} = c + 1/\bar{\alpha}$. This includes the prices firm 1 is charging after a deviation here.

Observation 2: Firm 1 cannot increase its profits by deviating to any p_{1L} with $p_{1L} \leq p_L^* - \frac{2\alpha_h - \alpha_\ell}{\alpha_h^2}$.

In this case, regardless of what prices are chosen at $t = 2$ firm 1 will sell at least as many units of good L as of good H . Hence, its profits are bounded above by the profits

from selling the same number of units at a price of $p_{1L} + w/\alpha_\ell$. If $p_{1L} + w/\alpha_\ell < 0$ then these profits are negative and not a profitable deviation. If $p_{1L} + w/\alpha_\ell > 0$ then profits are bounded above by the profits firm 1 would receive from selling to all consumers at this price. Given the assumed upper bound on p_{1L} the gain from the deviation is

$$\begin{aligned}\pi_1(p_{1L}) - \frac{1}{\bar{\alpha}} &\leq 2 \left(\frac{1}{\bar{\alpha}} - \frac{2\alpha_h - \alpha_\ell}{\alpha_h^2} \right) - \frac{1}{\bar{\alpha}} \\ &= \frac{2}{\alpha_\ell + \alpha_h} - 2 \frac{2\alpha_h - \alpha_\ell}{\alpha_h^2} = \frac{2}{\alpha_\ell + \alpha_h} \left(\left(\frac{\alpha_\ell}{\alpha_h} \right)^2 - \frac{\alpha_\ell}{\alpha_h} - 1 \right).\end{aligned}$$

This is negative when $\alpha_\ell/\alpha_h < \frac{1+\sqrt{5}}{2}$.

Observation 3: Firm 1 cannot increase its profits by deviating to any p_{1L} with $p_{1L} \geq p_L^* + \frac{2\alpha_h - \alpha_\ell}{\alpha_h^2}$.

In this case, firm 2 will make at least as many sales to low types as to high types. Hence, $p_{2H} = p_{2L} + w/\alpha_\ell = c + 1/\bar{\alpha}$. Again, part (a) of the proposition implies that the prices p_{1L} and p_{1H} firm 1 ends up charging cannot increase its profits.

QED

Proof of Lemma 1 To see that $p_{1U} = p_{2U} = w/\alpha_\ell$ is an equilibrium, note that when the firms are expected to set the same upgrade price, the mass of group j customers visiting firm 1 is $\frac{1}{2} + \frac{\alpha_j}{2}(p_{2L} - p_{1L})$. Profits are

$$\pi_1(w/\alpha_\ell, w/\alpha_\ell) = \sum_{j=1}^2 \left(\frac{1}{2} + \frac{\alpha_j}{2}(p_{2L} - p_{1L}) \right) (p_{1L} - c + w/\alpha_\ell).$$

Deviating to a lower upgrade price obviously cannot increase firm 1's profits – the lower price will not lead to any extra sales.

If firm 1 deviates to charge a higher price, no low types will purchase the upgrade. This decreases profits by $\left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_{2L} - p_{1L}) \right) \frac{w}{\alpha_\ell}$. Firm 1's sales to high types will be no higher. The upgrade price paid by these customers can be at most w/α_h . Hence the increase in profits on sales to high types is at most $\left(\frac{1}{2} + \frac{\alpha_h}{2}(p_{2L} - p_{1L}) \right) \left(\frac{w}{\alpha_h} - \frac{w}{\alpha_\ell} \right)$. The change in firm 1's profits from the deviation is thus bounded above by

$$\begin{aligned}&\left(\frac{1}{2} + \frac{\alpha_h}{2}(p_{2L} - p_{1L}) \right) \left(\frac{w}{\alpha_h} - \frac{w}{\alpha_\ell} \right) - \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_{2L} - p_{1L}) \right) \frac{w}{\alpha_\ell} \\ &= \frac{w}{2} \left[\left(\frac{1}{\alpha_h} - \frac{2}{\alpha_\ell} \right) + (p_{2L} - p_{1L}) \left(\frac{\alpha_h}{\alpha_h} - \frac{\alpha_h}{\alpha_\ell} - \frac{\alpha_\ell}{\alpha_h} \right) \right] \\ &\leq \frac{w}{2\alpha_h\alpha_\ell} \left[\alpha_\ell - 2\alpha_h - (p_{2L} - p_{1L})\alpha_h^2 \right].\end{aligned}$$

The bound on $|p_{2L} - p_{1L}|$ assumed in the lemma ensures that this is negative.

I now show that this is the only equilibrium.

First, note that the upper bound on the prices for L ensures that all consumers will visit one of the firms in equilibrium.

Next, note that in any equilibrium all firms choose p_{iU} equal to either w/α_h or $w\alpha_\ell$. To see this, one first shows that both firms must set $p_{iU} \geq w/\alpha_\ell$. Otherwise, the firm with the lower price attracts a positive mass of consumers. All of these consumers receive weakly higher ex ante expected utility from visiting that firm. Once they have sunk s visiting that firm they strictly prefer to buy there at the equilibrium prices. If the firm raises its upgrade price by some amount less than s/α_ℓ and keeps its price less than w/α_ℓ it will lose no sales. This would be a profitable deviation. The fact that $p_{iU} \geq w/\alpha_\ell$ implies that consumers in the low group get no surplus from buying the upgrade. Because of this and because the difference in prices for L is assumed to be bounded above by $(2\alpha_h - \alpha_\ell)/\alpha_h^2$, which is less than $1/\alpha_\ell$, each firm attracts a positive mass of consumers in any equilibrium. There cannot be an equilibrium with $w/\alpha_\ell < p_{iU} < w/\alpha_h$ because firm i would gain by raising its price slightly (if it is making any sales of good H) or by dropping its price to w/α_ℓ (if not). There cannot be an equilibrium with $p_{iU} > w/\alpha_h$ because firm i will sell no units of H , but would make positive sales by dropping its price to w/α_ℓ .

There cannot be an equilibrium with $p_{1U} = p_{2U} = w/\alpha_h$ because then the mass of customers from each group visiting firm 1 is exactly the same as when $p_{1U} = p_{2U} = w/\alpha_\ell$. The calculation above thus implies that firm 1 would increase its profits by deviating to $p_{1U} = w/\alpha_\ell$. To see that there can not be an equilibrium with $p_{1U} = w/\alpha_h$ and $p_{2U} = w/\alpha_\ell$ note that in this case the mass of low-type consumers visiting firm 1 would be exactly the same as in the above calculations, but that firm 1 would be visited by fewer high types. This makes the gain from deviating to $p_{1U} = w/\alpha_\ell$ even greater.

QED

Proof of Proposition 2

The result that $\bar{w} > \underline{w}$ follows from simple algebra:

$$\begin{aligned} \bar{w} > \underline{w} &\iff \frac{4\bar{\alpha}}{\sqrt{\alpha_\ell\alpha_h}} - 4 > \frac{\alpha_\ell - \alpha_h}{\alpha_\ell} \\ &\iff 4(\alpha_\ell + \alpha_h)^2\alpha_\ell^2 > \alpha_\ell\alpha_h(5\alpha_\ell - \alpha_h)^2 \end{aligned}$$

$$\iff \alpha_\ell(\alpha_\ell - \alpha_h)(4\alpha_\ell^2 - 13\alpha_\ell\alpha_h + \alpha_h) > 0.$$

This inequality is satisfied whenever $\frac{\alpha_\ell}{\alpha_h} > \frac{13+\sqrt{153}}{8} \approx 3.17$.

Another fact that will come in handy is that $\bar{w} < \frac{\alpha_\ell - \alpha_h}{\alpha_h}$. To see this, one can carry out a calculation similar to that above to show that

$$\frac{\alpha_\ell - \alpha_h}{\alpha_h} > \bar{w} \iff \alpha_h(\alpha_\ell - \alpha_h)(\alpha_\ell^2 + 3\alpha_\ell\alpha_h + 4\alpha_h) > 0.$$

(a) To show that the strategy profile where both firms set $p_{iL} = p_L^* \equiv c + 1/\alpha_\ell$ and $p_{iH} = p_H^* \equiv c + 1/\alpha_h$ is a sequential equilibrium (when combined with optimal behavior on the part of consumers) note first that the restrictions on w imply that when consumers anticipate that $p_{iL} = p_L^*$ and $p_{iH} = p_H^*$ then all consumers will visit the closest firm, low types will buy good L and high types will buy good H . (This follows from $\alpha_h(p_{iH} - p_{iL}) = \underline{w} < w$ and $\alpha_\ell(p_{iH} - p_{iL}) = (\alpha_\ell - \alpha_h)/\alpha_h > \bar{w} > w$). Hence, if the firms follow the given strategy profile each earns a profit of $\frac{1}{2\alpha_\ell} + \frac{1}{2\alpha_h}$.

If firm 1 deviates to any prices p_{1L} and p_{1H} at which it sells L to low types and H to high types and sells to some but not all of the customers in each market then its profits are

$$\pi_1(p_{1L}, p_{1H}) = \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_L^* - p_{1L}) \right) (p_{1L} - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_H^* - p_{1H}) \right) (p_{1H} - c).$$

This is a concave function uniquely maximized at $p_{1L} = \frac{1}{2}(c + p_L^* + 1/\alpha_\ell) = c + 1/\alpha_\ell$ and $p_{1H} = c + 1/\alpha_h$, so the deviation does not increase firm 1's profits.

If firm 1 sells L to low types and H to high types and sells to no or all customers in one (or both) markets then it is strictly worse off: zero sales earn zero rather than positive profits; and when selling to all customers of type j firm 1's profits from sales to type j consumers are no greater than the profits it would have earned from setting the price $p_{1j} = p_j^* - 1/\alpha_j$, and profits at this price are lower than the equilibrium profits because they are given by the formula above.

There is no profitable deviation which involves selling H to low types and L to high types because the high types will strictly prefer buying H whenever the low types are willing to buy H .

It is not necessary to check separately whether there is a profitable deviation involving selling only good L . If firm 1 has a profitable deviation which involved selling L at a price

of p_{1L} to a subset of the consumers, then it also has an even better profitable deviation in which it sells H at a price of $p_{1L} + w/\alpha_\ell - \epsilon$ to the same set of consumers.

To show that the profile given in (a) is an equilibrium it therefore remains only to show that there is no profitable deviation involving selling H to both populations. When firm 1 sells H to at least some of the consumers in each population at a price $p_{1H} > c$ its profits are bounded above by

$$\pi_1(p_{1H}) = \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_H^* - p_{1H}) \right) (p_{1H} - c) + \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_L^* - (p_{1H} - w/\alpha_\ell)) \right) (p_{1H} - c)$$

(The expression is only an upper lower bound and not necessarily the actual profit level because the quantity sold in each market is at most one.) This is a quadratic that is maximized at the unique solution to the first-order condition. Differentiating this expression we find after some algebra that it is maximized for

$$p_{1H} = c + \frac{1}{\bar{\alpha}} + \frac{w}{4\bar{\alpha}}.$$

Substituting into the profit function, the value at the maximum is $(1 + \frac{w}{4})^2 \frac{1}{\bar{\alpha}}$. This is no greater than the equilibrium profit if

$$\left(1 + \frac{w}{4} \right)^2 \frac{1}{\bar{\alpha}} \leq \frac{1}{2\alpha_\ell} + \frac{1}{2\alpha_h}.$$

This is satisfied for

$$w \leq 4 \left(\frac{\bar{\alpha}}{\sqrt{\alpha_\ell \alpha_h}} - 1 \right),$$

which is the assumption in the statement of the proposition that $w < \bar{w}$. This concludes the proof that the discriminatory profile described in part (a) of the proposition gives a sequential equilibrium.

To see that the standard pricing game sometimes has an equilibrium in which all consumers buy H at a price of $c+1/\bar{\alpha}$ (and that there are no other nondiscriminatory equilibria) consider the possibility of an equilibrium where all consumers buy good H . The same first-order analysis as in the proof of Proposition 1 shows that any equilibrium of this form would have to have $p_{1H} = p_{2H} = c + 1/\bar{\alpha}$. This profile will be an equilibrium if firm 1 cannot gain either by selling good H to the high types and nothing to the low types or by selling H to the high types and L to the low types. In the proof of Proposition 1, I noted that there is no profitable deviation involving only sales to the high types when $\alpha_\ell/\alpha_h < (3 + \sqrt{17})/2$

because at the price that maximizes profits from sales to the high types, the firm will sell to some low types as well. When α_ℓ/α_h is larger, firm 1's profit function does have a local maximum at $p_{1H} = c + \frac{1}{2\bar{\alpha}} + \frac{1}{2\alpha_h}$. Firm 1's profit when it sets this price and sells to only high types is $\frac{\alpha_h}{2} \left(\frac{1}{2\bar{\alpha}} + \frac{1}{2\alpha_h} \right)^2$. This is larger than $\frac{1}{\bar{\alpha}}$ only if $\alpha_\ell/\alpha_h > 5 + \sqrt{32} \approx 10.66$. Hence, for the parameter values of the proposition, this deviation is not profitable. In the proof of Proposition 1, the optimal deviation involving selling both H and L could take any of three forms. Given the restriction on w in Proposition 2, only the second of these (corresponding to observation 6 in the earlier proof) arises and the optimal deviation of this form is $p_{1L} = c + \frac{1}{2\bar{\alpha}} + \frac{1-w}{2\alpha_\ell}$ and $p_{1H} = c + \frac{1}{2\bar{\alpha}} + \frac{1}{2\alpha_h}$. The profit from this deviation is

$$\frac{\alpha_h}{2} \left(\frac{1}{2\bar{\alpha}} + \frac{1}{2\alpha_h} \right)^2 + \frac{\alpha_\ell}{2} \left(\frac{1}{2\bar{\alpha}} + \frac{1-w}{2\alpha_\ell} \right)^2$$

A numerical calculation shows that this deviation is profitable if $\alpha_\ell/\alpha_h < 6.3$ and w is close to \underline{w} . When w is close to \bar{w} (and for all $w \in (\underline{w}, \bar{w})$ when $\alpha_\ell/\alpha_h > 6.4$) the deviation is not profitable and hence there is a nondiscriminatory equilibrium.

To see that there are no other symmetric sequential equilibria in the add-on pricing game, the only additional possibility that needs to be checked is whether there is an equilibrium in which each firm sells L to the low types and H to the high types. There can be no such equilibrium with both types strictly preferring to buy the good they are buying because then the first order conditions for each firm not wanting to raise or lower each price (used in the existence argument) imply that the equilibrium must have $p_{iL} = c + 1/\alpha_\ell$ and $p_{iH} = c + 1/\alpha_h$. There can be no such equilibrium in which the low types are indifferent to buying H because in that case firm 1 would profit from lowering the price of the upgrade by ϵ and selling it to the low types as well. There can be no such equilibrium in which the high types are indifferent to buying L because (as in the proof of Proposition 1) considering the first order condition for firm 1 deviating and raising or lowering both p_{1L} and p_{1H} by exactly the same amount the only possible equilibrium of this form would be to have $p_{1L} = c + 1/\bar{\alpha} - w/2\bar{\alpha}$ and $p_{1H} = c + 1/\bar{\alpha} - w/2\bar{\alpha} + w/\alpha_h$. This is not an equilibrium because firm 1 could increase its profits by raising p_{1L} slightly. Such a change does not affect firm 1's sales to high types. In the low market firm 1's profits (in a neighborhood above $c + 1/\bar{\alpha} - w/2\bar{\alpha}$) are

$$\left(\frac{1}{2} + \frac{\alpha_\ell}{2} \left(c + \frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} - p_{1L} \right) \right) (p_{1L} - c).$$

The derivative of this expression with respect to p_{1L} evaluated at $c + 1/\bar{\alpha} - w/2\bar{\alpha}$ is

$$\frac{1}{2} \left(1 - \alpha_\ell \left(\frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} \right) \right).$$

This is positive if $w > \underline{w}$.

(b) Consider now the add-on pricing game. Suppose that in a sequential equilibrium both firms set $p_{iL} = p_L^*$ at $t = 1$. The first thing to note is that at $t = 2$ the optimal continuation equilibrium for the firms involves the add-on being now sold for a price of w/α_h (both in equilibrium and following small deviations).

Claim: If $|p_{1L} - p_L^*| < 1/\alpha_h$ and $p_{2L} = p_L^*$ then there is a sequential equilibrium in which both firms choose $p_{iU} = w/\alpha_h$ at $t = 2$. This is the best equilibrium for the firms.

To see this note again that because of the structure of the consumer search problem the only possible equilibrium upgrade prices will be w/α_ℓ and w/α_h . If both firms set $p_{iU} = w/\alpha_h$, then at $t = 2$ the firm that chose a lower price at $t = 1$ will be visited by at least half of the low types and by at most all of the low types. Hence, at least one-third of the consumers visiting the low priced firm are high types and the assumption of the proposition that $w/\alpha_h > 3w/\alpha_\ell$ ensures that this firm is better off selling to just the high types. The firm that set the higher price at $t = 1$ will be visited by more high types than low types and is thus also better choosing the high upgrade price.

If firm 1 deviates from the equilibrium and chooses a price p_{1L} with $|p_{1L} - p_L^*| < 1/\alpha_\ell$ and the firm-optimal continuation equilibrium is played at $t = 2$ then firm 1's profits are

$$\pi_1(p_{1L}) = \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(p_L^* - p_{1L}) \right) (p_{1L} - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_L^* - p_{1L}) \right) \left(p_{1L} + \frac{w}{\alpha_h} - c \right).$$

This is a quadratic maximized at the solution to the first-order condition. The derivative is

$$\frac{d\pi_1}{dp_{1L}} = 1 - 2\bar{\alpha}p_{1L} + \bar{\alpha}p_L^* + \bar{\alpha}c - w/2.$$

Setting $p_{1L} = p_L^*$ and solving we see that the only possible symmetric equilibrium of this form is $p_L^* = c + 1/\bar{\alpha} - w/2\bar{\alpha}$. This completes the proof of the uniqueness claim of the proposition.

The calculation above also implies that no deviation from this profile with $|p_{1L} - p_L^*| < 1/\alpha_\ell$ will increase firm 1's profits. To complete the proof that this is indeed an equilibrium

one needs to verify that larger deviations (for which the expression above is not the correct profit function) also do not increase firm 1's profits.

To see that no deviation to a price $p_{1L} > p_L^* + 1/\alpha_\ell$ can increase firm 1's profits, note that for prices in this range firm 1's profits (if they are nonzero) are given by

$$\pi_1(p_{1L}) = \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_L^* - p_{1L}) \right) \left(p_{1L} + \frac{w}{\alpha_h} - c \right).$$

The derivative of this expression is

$$\frac{d\pi_1}{dp_{1L}} = \frac{1}{2} - \alpha_h p_{1L} + \frac{\alpha_h}{2} p_L^* + \frac{\alpha_h}{2} c - \frac{w}{2}.$$

The derivative is decreasing in p_{1L} and after some algebra one can show that it is negative when evaluated at $p_L^* + 1/\alpha_\ell$ when $w \geq \underline{w}$. Hence, profits from any deviation in this form are less than the profits from a deviation to $p_{1L} = c + 1/\alpha_\ell$, which are less than the putative equilibrium profit by the above argument. (Apart from the algebra the result in this case should also be obvious: firms are keeping p_{1L} and p_{1H} farther apart than is optimal. It would make no sense to increase the already too-high price in market H and abandon market L .)

To see that there is no profitable deviation with $p_{1L} < p_{2L} - 1/\alpha_h$ note that with such a price firm 1 sells to all of the low and high type consumers. (There cannot be an equilibrium where firm 2 attracts some high types by charging a low upgrade price because firm 2 will attract no low types and hence would always raise its upgrade price by s once consumers visit it.) Its profits are bounded above by $(p_L^* - 1/\alpha_h - c) + (p_L^* - 1/\alpha_h + w/\alpha_h - c)$. This is less than the equilibrium profit of $p_L^* + w/2\alpha_h - c$ if

$$\begin{aligned} p_L^* - c < \frac{2}{\alpha_h} - \frac{w}{2\alpha_h} &\iff \frac{2-w}{2\bar{\alpha}} < \frac{4-w}{2\alpha_h} \\ &\iff w < \frac{4\alpha_\ell}{\alpha_\ell - \alpha_h}. \end{aligned}$$

The restrictions that $w < \bar{w}$ and $\alpha_\ell/\alpha_h < 10$ imply that the left hand side is less than four. The right hand side is always greater than four, so the deviation is never profitable.

Finally, to see that there is no profitable deviation with $p_{1L} \in (p_L^* - \frac{1}{\alpha_n}, p_L^* - \frac{1}{\alpha_\ell})$, note that firm 1's profits with such a price are

$$\pi_1(p_{1L}) = (p_{1L} - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2}(p_L^* - p_{1L}) \right) \left(p_{1L} + \frac{w}{\alpha_h} - c \right).$$

The profits from such a deviation cannot be profitable if this expression does not have a local maximum in the interval because we've already seen that deviations to either endpoint of the interval are not profitable. The solution to the first order condition for maximizing the expression above is

$$p_{1L} = c + \frac{3}{2\alpha_h} + \frac{1}{2\bar{\alpha}} - \frac{w}{4\bar{\alpha}} - \frac{w}{2\alpha_h}.$$

This fails to be interior if

$$c + \frac{3}{2\alpha_h} + \frac{1}{2\bar{\alpha}} - \frac{w}{4\bar{\alpha}} - \frac{w}{2\alpha_h} > c + \frac{1}{\bar{\alpha}} - \frac{w}{2\bar{\alpha}} - \frac{1}{\alpha_\ell}.$$

After some algebra one can see that this is the case whenever

$$3 + 3\frac{\alpha_h}{\alpha_\ell} + 2\frac{\alpha_h^2}{\alpha_\ell^2} > w,$$

which is true for all $w < \bar{w}$ as long as $\alpha_\ell/\alpha_h < 10$ because the left hand side is at least 3.32 and the right hand side is at most $4(5.5/\sqrt{10} - 1) \approx 2.96$. Hence, the deviation cannot be profitable. (The assumption of the proposition that $\alpha_\ell/\alpha_h < 10$ could be weakened by computing the profits at the interior optimum when it exists and showing that they remain below the equilibrium profit level for a broader range of parameter values.)

The first-order analysis at the start of the proof of part (b) of the proposition established that no other first-period prices are possible in a symmetric equilibrium in which the firms sell the upgrade at a price of w/α_h on the equilibrium path and after any small deviation from the first-period equilibrium prices. This does not, however, imply that there are no other equilibria.

To see that the equilibrium of part (b) of Proposition 1 can be resurrected for at least some of the parameter values covered under Proposition 2, note that if consumers' beliefs are that the firms set $p_{iL} = c + 1/\bar{\alpha} - w/\alpha_\ell$ at $t = 1$ and then set $p_{iU} = w/\alpha_\ell$ on the equilibrium path and after nearby deviations, then if firm 1 raises its upgrade price at all at $t = 2$, all low types who visit will refuse to buy the upgrade and some high types will decide to purchase nothing and visit firm 2 at $t = 4$. When s is small firm 1's profits will be approximately equal to

$$\pi_1(p_{1L}, p_{1H}) = \left(\frac{1}{2} + \frac{\alpha_\ell}{2}(c + 1/\bar{\alpha} - w/\alpha_\ell - p_{1L}) \right) (p_{1L} - c) + \left(\frac{1}{2} + \frac{\alpha_h}{2}(c + 1/\bar{\alpha} - p_{1H}) \right) (p_{1H} - c).$$

This is precisely the expression we considered when assessing whether in the standard pricing game there was any profitable deviation from a profile which sold good H to all consumers at a price of $c + 1/\bar{\alpha}$. The result of part (a) of the proposition implies that the deviation we are considering here cannot be profitable.

To construct equilibria with other profits levels, one could for example, suppose that firms 1 and 2 both set $p_{iL} = c + \frac{1}{\alpha} - \frac{w}{2\alpha} + \epsilon$ at $t = 1$ and at $t = 2$ set $p_{iU} = w/\alpha_h$ if there was no deviation and $p_{iU} = w/\alpha_\ell$ if there was a deviation. The calculations above imply that any deviation at $t = 1$ would produce at most an $O(\epsilon)$ increase in profits if the firms charged w/α_h at $t = 2$. When firms switch to the lower upgrade price they incur a discrete loss of about $w/2\alpha_h - w/\alpha_\ell$, and hence the net change in profits from the deviation is negative.

QED