

Dynamic price competition with capacity constraints and strategic buyers

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Abstract

We analyze a set of simple dynamic models where sellers are capacity constrained over the length of the model. Buyers act strategically in the market, knowing that their purchases will affect future prices. The model is examined when there are single and multiple buyers, with both linear and non-linear pricing. We find that, in general, there are only mixed strategy equilibria and that sellers get a rent above the amount needed to satisfy the market demand that the other seller cannot meet. Buyers would like to commit not to buy in the future. Furthermore, buyers have an incentive to vertically integrate and buy one of the suppliers, even though this will make the other supplier a monopolist. Also, sellers' market shares may be quite asymmetric even though they are ex-ante identical.

1 Introduction

In many durable goods markets, sellers who have market power and intertemporal capacity constraints face strategic buyers who make purchases over time to match their demands. There may be a single buyer, as in the case of a government that purchases military equipment or awards construction projects, such as for bridges, roads, or airports, and chooses among the offers of a few large available suppliers. Or, there may be a small number of large buyers, such as in the case of airline companies that order aircraft or that of shipping companies that order cruise ships, where the supply could come only from a small number of large, specialized companies.¹ The capacity constraint may be due to the production technology: a construction company that undertakes to build a highway today may not have enough engineers or machinery available to compete for an additional large project tomorrow, given that the projects take a long time to complete; a similar constraint is faced by an aircraft builder that accepts an order for a large number of aircraft. Or, the capacity constraint may simply correspond to the flow of a resource that cannot exceed some level within a given period: thus, if a supplier receives a large order today, he may not be able to supply additional quantity tomorrow. This effect may be indirect, if the resource is a necessary ingredient for a final product, with no substitutes (as often in the case of pharmaceuticals). Cases like the ones mentioned above suggest a need to study dynamic oligopolistic price competition under capacity constraints, when buyers are also strategic. A key consideration is that when a buyer purchases a larger quantity from a seller in the current period, the available quantity that seller could supply to the market subsequently is lower. Although this topic is both important and interesting, it has not been treated yet in the literature.

To obtain some first insights into the problem, consider the following simple setting. Take two sellers of some homogeneous product, say aircraft, to fix ideas. Each seller cannot supply more than a given number of aircraft over two periods. Suppose that there is only one large buyer in this market, this may be the defense department, with a demand that exceeds the capacity of each

¹Anton and Yao (1990) provide a critical survey of the empirical literature on competition in defense procurement - see also Burnett and Kovacic (1989) for a discussion of the key issues and evaluation of policies. In an empirical study of the defense market, Greer and Liao (1986) find (p.1259) that “the aerospace industry’s capacity utilization rate, which measures propensity to compete, has a significant impact on the variation of defense business profitability and on the cost of acquiring major weapon systems under dual-source competition”. Ghemawat and McGahan (1998) show that order backlogs, that is, the inability of manufacturers to supply products at the time the buyers want them, is important in the U.S. large turbine generator industry and affects firms’ strategic pricing decisions. Also large cruise ships can take three years to build a single ship and an additional two years and three months to produce another one.

seller but not that of both sellers combined. Let the period one prices be lower for one seller than another. Then, if the buyer's purchases exhaust the capacity of the low priced seller, only the other seller will remain active in the second period and, unconstrained from any competition, he will charge the monopoly price. A number of questions arise. Anticipating such behavior, how should the buyer behave? Should he split his orders in the first period, in order to preserve competition in the future, or should he get the best deal today? Given the buyer's possible incentives to split orders, how will the sellers behave in equilibrium? Should sellers price in a way that would induce the buyer to split or not to split his purchases between the sellers? How do sellers' equilibrium profits compare with the case of only a single pricing stage? Does the buyer have an incentive to commit to not making purchases in the future? Are there incentives for the buyer to vertically integrate with a seller?

An additional set of questions emerges when there is more than one buyer. Suppose there are two sellers and two buyers. Again, if one buyer places a large order with one seller, then the remaining seller may be unconstrained by competition in period two. Would the other buyer attempt to balance the situation? Would the buyers like to coordinate their purchases by targeting one seller or both? Also, what happens as the number of buyers increases and, thus, the incentive of each of them to act strategically diminishes?

We consider a set of simple dynamic models with the following key features. There are two sellers, each with fixed capacity over two periods. Sellers set first period prices and then buyers decide how many units they wish to purchase from each seller. The situation is repeated in the second period, given the remaining capacity of the firms; sellers set prices and buyers decide which firm to purchase from. We examine the case of a single and multiple buyers. In each case, we consider both linear as well as non-linear pricing.

Our main results are as follows. Under monopsony and linear pricing, a pure strategy subgame perfect equilibrium fails to exist. This is due to two phenomena. First, the buyer's incentive to split her orders if the prices are close. This gives the sellers incentives to raise price. On the other hand, if prices get "high", then sellers have incentives to lower their prices, and sell all their capacity. We characterize mixed strategy equilibria and show that the buyer may have a strict incentive to split his orders. Also, the sellers make a positive economic rent above the profits of serving the buyer's residual demand, if the other seller sold all of his units. Furthermore, the buyer has a strict incentive to vertically integrate, even though the other firm will be a monopolist. Finally, for high discount factors the buyer would like to commit to not make purchases in the second period, so

as to induce strong price competition in the first period. This last result is consistent with the practice in the airline industry, where airliners have options to buy airplanes in the future.

Under non-linear pricing, we show that the ability of each seller to price each of his units separately allows us to derive a pure strategy equilibrium where the sellers make no rents, the buyer does not have an incentive to vertically integrate, and does not have an incentive to commit not to buy in the future. These results are due to the fact that the seller's profitability on each unit can be separated with non-linear pricing.

In the oligopsony case, we find that it is now the buyers that must play a mixed strategy, randomizing between which of the two sellers they should buy from, while there does not have to be the possibility of splitting orders in equilibrium. The reason that buyers must mix in an oligopsony, while sellers must mix in monopsony, is that there is no one that a buyer can miscoordinate with in a monopsony, but in an oligopsony in equilibrium buyers must not coordinate their purchases. We also find that sellers make positive rents if buyers cannot commit not to buy in the future and that buyers have an incentive to vertically integrate. These results hold under both linear and non-linear pricing. Finally, we demonstrate that *ex post* market shares and profits of sellers may be quite different, even though the firms are *ex ante* identical.

Our paper is related to three distinct literatures. First, to the literature on pricing with capacity constraints. It is already known from the classic work of Edgeworth (1897) that capacity constraints may dramatically alter the nature of price competition in oligopolistic markets, possibly leading to a “nonexistence of equilibrium” or, as sometimes described “cycles”.² Mixed strategy pricing equilibria under capacity constraints are derived in Levitan and Shubik (1972) and Osborne and Pitchik (1986).³ In a static model, Lang and Rosenthal (1991) characterize mixed strategy price equilibria in a game where sellers (“contractors”) face increasing cost for each additional unit they supply. Dudey (1992) derives the price equilibrium when capacity-constrained sellers face buyers that arrive to the market sequentially. There is a well-known literature on firms that choose their capacities, in anticipation of an oligopoly competition stage; see e.g. Dixit (1980), Spulber (1981), Kreps and Scheinkman (1984), Davidson and Deneckere (1986a), Deneckere and Kovenock (1996), and Allen, Deneckere, Faith, and Kovenock (2000). A number of papers have also analyzed the effect of capacity constraints on collusion - see e.g. Brock and Scheinkman (1985), Davidson and

²Maskin and Tirole (1988) provide game theoretic foundations for “Edgeworth cycles” in a somewhat different setting, without capacity constraints.

³Kirman and Sobel (1974) prove equilibrium existence in a dynamic oligopoly model with inventories.

Deneckere (1986b), Lambson (1987, 1994), Rotemberg and Saloner (1989), Compte, Jenny and Rey (2002), and Kovenock and Dechenaux (2003). Relative to these papers, a crucial difference in our analysis is that we consider strategic behavior also on the buyers' side and also sellers have intertemporal capacity constraints. In particular, we examine how the sellers' capacities evolve over time, as interrelated with their pricing strategies and the buyers' decisions.

The second set of papers that our work is related to are when buyers influence the degree of competition among (potential) suppliers, in particular on buyers' incentives to act strategically when facing competing sellers, as in the context of "split awards" and "dual-sourcing". Rob (1986) studies procurement contracts that would allow selection of an efficient supplier, while also providing incentives for product development. Anton and Yao (1987, 1989, 1992) consider models with two sellers and a single buyer, where the buyer can buy either from one seller or split his order and buy from each seller. They find conditions under which a buyer will split his order and find that there can be seemingly collusive equilibria. Our work differs in two important ways. The intertemporal links are at the heart of our analysis: the key issue is how purchasing decisions today affect the sellers' remaining capacities tomorrow. In contrast, the work mentioned above focuses on static issues and relies on cost asymmetries. Related studies on dual-sourcing are offered by Riordan and Sappington (1987) and Demski, Sappington and Spiller (1987). Strategic purchases from competing sellers and a single buyer in a dynamic setting are also studied under "learning curve" effects; see e.g. Riordan and Cabral (1994) and Lewis and Yildirim (2002). One difference that should be noted is that in our case by buying a larger quantity from one seller you make that seller less competitive in the following period (at the extreme case: inactive) - in the learning curve case, the more you buy from a seller, the more competitive you make him, as his unit cost decreases.⁴ Finally, our analysis has implications related to vertical integration strategies is thus related to the literature on the issue (see e.g. Ma, 1997, for a related analysis of vertical integration under option contracts, and Rey and Tirole, 2003, on foreclosure).

A third set of papers that our work is related to is on bilateral oligopolies, where both the sellers and the buyers are large players and act strategically. While in many important markets players have significant power on both sides of the market, such situations have not generally received

⁴Bergemann and Välimäki (1996, 2000) examine models where in each period sellers set prices and buyers choose which seller to purchase from. Buyers' decisions affect how competitive each seller could be in subsequent periods, however this is in a very different setting where the action comes from experimentation and learning, not from capacities.

enough attention.⁵ In our analysis, with two sellers, we examine both the case of a single and that of multiple buyers and we emphasize that the sellers' behavior changes qualitatively when we move from the former case to the latter.

The remainder of the paper is organized as follows. The model is set up in Section 2. Section 3 characterizes the equilibrium with one buyer and linear pricing. In Section 4 we characterize the equilibrium with one buyer and non-linear pricing. The duopsony case is analyzed in Section 5, first under linear and then under non-linear prices. Section 6 extends the analysis to the case of many buyers. We conclude in Section 7.

2 The basic model

The game lasts two periods. There are $N + 2$ firms, two sellers and N buyers. The product is perfectly homogeneous and the sellers are identical. Each seller has a capacity to produce, for the two periods, a total of $2N$ units at marginal cost of 0.⁶ The goods are durable over the lifetime of the model. Each buyer values a first unit at V_1 and a second unit at V_2 in each of the periods and a third unit at V_3 in period 2.⁷ We assume that $V_1 > V_2 > V_3 > 0$.

In each period, each of the sellers sets a price for each of his available units of capacity. We consider competition both with linear and with non-linear pricing. For simplicity, we assume non-discriminatory pricing by sellers.⁸ Each buyer chooses how many units he wants to purchase from each seller at the price specified, as long as the seller has enough capacity. If the demand by buyers is greater than a seller's capacity, then they are rationed. The rationing rule that we use is that each buyer is equally likely to get his order filled. The rationed buyer can buy from the other seller as many units as they want. We assume that sellers commit to their prices.⁹ All information is common knowledge and symmetric. All firms have a common discount factor δ . In each case

⁵Recent papers that study various aspects of bilateral oligopoly include Horn and Wolinsky (1988), Dobson and Waterson (1997), Bloch and Ghosal (1997), Hendricks and McAfee (2000), Björnerstedt and Stennek (2001) and Inderst and Wey (2003).

⁶This is independent of when these units are supplied during the two periods.

⁷We could allow the demand of the third unit could be random. The qualitative features of our results would not change.

⁸Clearly, this assumption does not matter when there is a single seller. The flavor of our results would be the same if discriminatory pricing was allowed with multiple buyers.

⁹Our results would not change qualitatively if sellers could choose which player to ration.

examined, we are looking for a symmetric subgame perfect equilibrium.¹⁰

The interpretation of the timing of the game is completely straightforward in case the sellers' supply comes from an existing stock (either units that have been already produced, or some natural resource). In case there is production taking place in every period, there may be more than one possible interpretations of the intertemporal capacity constraint, depending on the details of the technology. One simple way to understand the timing, in such a case, is illustrated in Figure 1. The idea here is that actual production takes time. Thus, orders placed in period are not completed before period two orders arrive. Since each seller has the capacity to only work on a limited number of units at a time, units ordered in period one restrict how many units could be ordered in period two. In such a case, since our interpretation involves delivery after the current period, the buyers' values specified in the game should be understood as the present values for these future deliveries (and the interpretation of discounting should be also accordingly adjusted.)

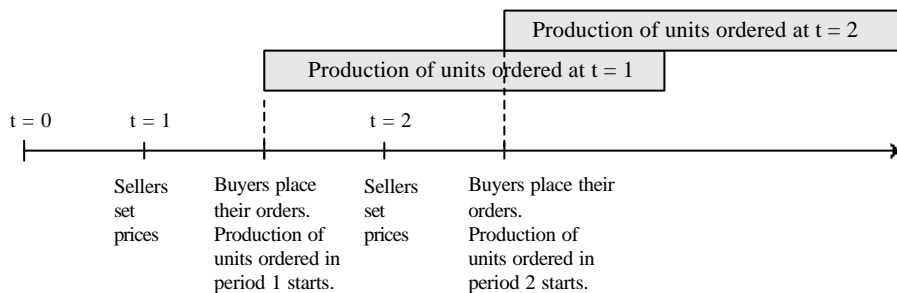


Figure 1: Timing

3 Monopsony with linear pricing

We first examine the single buyer case ($N = 1$), that is, monopsony and consider competition when the two sellers are restricted to linear pricing. We then allow for non-linear pricing under monopsony in the next section.

We are constructing a subgame perfect equilibrium, and thus we work backwards by starting from period 2.

¹⁰Let us observe, before proceeding to the analysis, that negative prices cannot be part of an equilibrium. Suppose in equilibrium some seller charged a negative price in some period. Then, either a buyer would have a strict incentive to buy all the available units of that seller or would choose to wait and purchase those units at a later time if the relevant price was expected to be even lower. Either way, this seller could do better by increasing his price to zero, thus increasing his profit from a negative level to zero. This observation allows us to simplify the presentation of the arguments, by focusing on non-negative prices.

3.1 Second period

There are several cases to consider, depending on how many units the buyer has bought from each seller in period one.

Buyer bought two units in period 1. If the buyer bought a unit from each of the sellers in period 1, then the price in period 2 would be 0 due to Bertrand competition. If the buyer bought both units from the same firm, then the other firm would be a monopolist in period 2 and charge V_3 . Thus, period 2 equilibrium profit of a seller that has one remaining unit of capacity is 0 and that of a seller with two remaining units of capacity is V_3 .

Buyer bought one unit in period 1. The buyer has demand for two units, one of the sellers has capacity of 1 unit, say seller 1, while the other has a capacity of 2 units, seller 2. We demonstrate that there is no pure strategy equilibrium in period 2 by the following Lemma.

Lemma 1 *If the buyer bought one unit in period 1 in the linear pricing monopsony model, then there is no pure strategy equilibrium in period 2.*

P proof. First, notice that the equilibrium cannot involve seller 2 charging a zero price: that seller could increase his profit by raising his price (as seller 1 does not have enough capacity to cover the buyer's entire demand). Thus, seller 1 would also never charge a price of zero. Suppose now that both sellers charged the same positive price. One, if not both, sellers have a positive probability of being rationed. A rationed seller could defect with a slightly lower price and raise his payoff. Suppose that the prices are not equal: $p_i < p_j \leq V_3$. Then, the buyer would buy at least one unit from seller i and would buy a second unit from this seller if he has two units of capacity. Clearly, seller i could increase his payoff by increasing his price since he still sells the same number of units. Similarly, seller i can improve his payoff by increasing his price if $p_i < V_3 \leq p_j$. Finally, if $V_3 \leq p_i < p_j$, seller j makes 0 profit and can raise his payoff by undercutting firm i 's price. ■

There is a unique mixed strategy equilibrium which we provide in the following Lemma.

Lemma 2 *If the buyer bought one unit in period 1 in the linear pricing monopoly model, then there is a unique mixed strategy equilibrium where sellers mix on the interval $[V_3/2, V_3]$ with seller 1 having an expected profit of $V_3/2$ and seller 2 having an expected profit of V_3 .*

P proof. First, we argue that the players choose prices in the interval $[\frac{V_3}{2}, V_3]$. Suppose that seller 2, asked a price p less than $V_3/2$. If seller 1 charges a price less than p , then seller 2 will sell 1

unit, while if seller 1 charges a price higher than p , seller 2 sells 2 units. Seller 2 could improve his payoff no matter what prices seller 1 asks by asking for $V_3 - \epsilon$ for ϵ very small and selling at least one unit for sure, since $V_3 - \epsilon > 2p$. Since seller 2 will charge a price of at least $V_3/2$, then so will seller 1; otherwise, seller 1 could increase his price and still guarantee a sell of 1 unit. Thus, both sellers charge at least $V_3/2$. Now, we argue that price will be no more than V_3 . Take the highest price p offered in equilibrium greater than V_3 . First, assume that there is not a mass point by both sellers at this price. This offer will never be accepted by the buyer, since he will always buy the second unit from the lower priced seller and his valuation for a third unit is $V_3 < p$. The seller could always improve his payoff by charging a positive price less than $V_3/2$. Second, if there is a mass point by both sellers, then at least one of them is rationed with positive probability and a seller can slightly undercut his price and improve his payoff. Thus, all prices will be between $V_3/2$ and V_3 .

Now, we argue that the expected equilibrium period 2 payoffs are $V_3/2$ for seller 1 and V_3 for seller 2. Given that the equilibrium prices are between $V_3/2$ and V_3 , we know that the profits for seller 1 is at least $V_3/2$ and for seller 2 at least V_3 . First, we argue that it can never be the case that *both* sellers will have an atom at the highest price p_H ; later we further show that seller 2 will have a mass point at p_H . If both did, then there is a positive probability of a seller being rationed, and a seller could improve his payoff by slightly lowering his price. Thus, a seller asking p_H knows that he will be the highest priced seller. If he is seller 1 he will not make a sell, while if he is seller 2 he will make a sell of one unit. If seller 2 charges p_H he knows that his payoff will be p_H , thus p_H must equal V_3 . If the lowest price offered in equilibrium, p_L , were greater than $V_3/2$, then seller 2 could improve his payoff by offering $p_L - \epsilon > V_3/2$, with the buyer buying two units from the seller and thus improve his payoff above V_3 . Thus, the lowest price is $V_3/2$. Since both sellers must offer this price, seller 1's expected payoff must be $V_3/2$.

We now find the equilibrium price distributions. Let F_i be the distribution of seller i 's price offers. Seller 1's price distribution is then determined by indifference for seller 2:

$$p [F_1(p) + 2(1 - F_1(p))] = V_3, \quad (1)$$

since seller 2's expected payoff is V_3 by the earlier argument. Seller 2's payoff is calculated as follows. When seller 2 charges price p , then with probability $F_1(p)$ seller 1's price is lower and seller 2 sells one unit, while with probability $1 - F_1(p)$ seller 1's price is higher and seller 2 sells both his units. Solving (1), we obtain:

$$F_1(p) = 2 - \frac{V_3}{p}. \quad (2)$$

Seller 2's price distribution is a little more complicated. For $p < V_3$, it is determined by

$$p[1 - F_2(p)] = \frac{V_3}{2}. \quad (3)$$

Seller 1 sells one unit if his price is lower than the rival's and this happens with probability $1 - F_2(p)$; otherwise, he sells no units. This equals seller 1's expected profit $V_3/2$ by Lemma 2. Condition (3) implies

$$F_2(p) = 1 - \frac{V_3}{2p}. \quad (4)$$

There is a mass of $1/2$ at price V_3 (see Figure 2). Simple arguments can be used to establish that the equilibrium pricing distributions must be continuous and that the only mass point may be located at V_3 for seller 2. ■

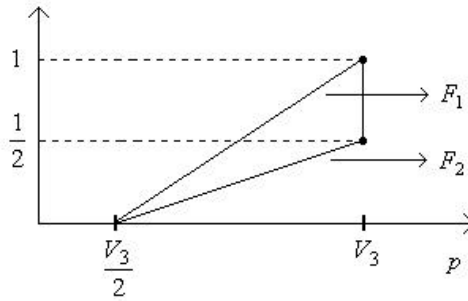


Figure 2: Mixed strategy equilibrium.

The seller with two units of capacity can always guarantee himself a payoff of at least V_3 , since he knows that, no matter what the other seller does, he can always charge V_3 and sell at least one unit. This is the high-capacity seller's *security profit level*, π_H^S . In general, if the buyers have value for B units in period 2, and the capacity of the low-capacity seller in period 2 is C , then the high-capacity seller's security profit, is $\pi_H^S = V_3(B - C)$.¹¹ The high-capacity seller's security profit puts a lower bound on the price offered in period 2. In the situation of Lemma 2, the lowest price is $V_3/2$, since the seller will never charge a lower price because he can at most sell two units and does better by selling one unit at V_3 . This puts a lower bound of $V_3/2$ on the period 2 profit of seller 1, the low-capacity seller; this is the low-capacity seller's security profit, π_L^S . In general, the high-capacity seller will never charge a price below $V_3(B - C)/B$, since a lower price will lead to profit lower than his security profit. Thus, the low-capacity seller's security profit is $\pi_L^S = C \frac{V_3(B - C)}{B}$. Competition between the two seller's fixes their profits at their security levels.

¹¹It easy to show that B is always at least as large as C .

Buyer bought no units in period 1. Each seller enters period 2 with 2 units of capacity, while the buyer demands 3 units. Using arguments similar to the ones in Lemma 1, we conclude that there is no pure strategy equilibrium. There is a unique symmetric mixed strategy equilibrium. Each player's expected second-period equilibrium payoff is V_3 ; this is the security profit of each seller. Using arguments similar to those in Lemma 1, the players mix on the interval $\left[\frac{V_3}{2}, V_3\right]$, with no mass points or gaps. The buyer's demand is always met and he buys two units from the lowest priced seller and one from the highest priced seller. The sellers' distribution of prices satisfies

$$p [F(p) + 2(1 - F(p))] = V_3, \quad (5)$$

or

$$F(p) = 2 - \frac{V_3}{p}. \quad (6)$$

The indifference condition underlying this expression is similar to condition (1) for a seller with two units in the subgame examined before.

The equilibrium behavior in the second period is now summarized:

Lemma 3 *Second period competition for a monopsonist facing linear pricing falls into one of three categories. (i) If only one seller is active (the rival has zero remaining capacity), that seller sets the monopoly price, V_3 , and extracts the buyer's entire surplus. (ii) If each seller has enough capacity to cover by himself the buyer's demand then there is (Bertrand) pricing at zero. (iii) If the buyer's demand exceeds the capacity of one seller but not the aggregate sellers' capacity, then there is no pure strategy equilibrium. In the mixed strategy equilibrium a seller with two units of capacity has expected profit equal to V_3 and a seller with one unit of capacity has expected profit equal to $V_3/2$.*

Case (i) in the above Lemma occurs when the buyer bought two units from the same seller in period 1; then the seller that has remaining capacity sets his price equal to V_3 . Case (ii) occurs when the buyer bought one unit from each seller in period 1. Case (iii) occurs when the buyer bought one unit in period 1 or when the buyer bought no units in period 1.

3.2 First period

Now, let us go back to period 1. First, we begin with the buyer's decision. Suppose that the prices in period 1 are p_H and p_L , with $p_H \geq p_L$. Note that pricing in period one could in principle be done through either pure or mixed strategies. In the former case, p_H and p_L are the prices set by

the two sellers, whereas in the latter these are realizations of the strategies. Further let us denote the expected period 2 price of seller i if he has k units of capacity by $E p_k^i$. We use Lemma 3 in computing the payoffs.

The buyer's payoff if he buys one unit from each of the firms in period 1 is

$$W_1 \equiv (1 + \delta)(V_1 + V_2) - p_H - p_L + \delta V_3.$$

The buyer gets one unit for free in the following period (since competition drives the price to zero).

The buyer's payoff if he buys both units from firm L is

$$W_2 \equiv (1 + \delta)(V_1 + V_2) - 2p_L.$$

In this case, the buyer faces a monopolist and pays V_3 in period 2.

The buyer's expected payoff if he buys only one unit from firm L is

$$W_3 \equiv (1 + \delta)V_1 - p_L + \delta [V_2 + V_3 - E \min[p_1, p_2] - E p_2]$$

In the following period the buyer will buy two additional units. He will pay the lowest price offered in the following period for the second unit and will buy the third unit from the seller who has two units of capacity in period 2, since either he is the low priced seller or the other seller has no more capacity.

The buyer's expected payoff if he buys no units in period 1 is

$$W_4 \equiv \delta [V_1 + V_2 - 2E \min[p_2^A, p_2^B] + [V_3 - E \max[p_2^A, p_2^B]]]$$

The buyer will buy three units in the next period. He will buy two units from the low priced seller and one from the higher priced seller, where each seller, A or B , has two units of capacity in the next periods.

The buyer's expected payoff if he buys two units from the lowest priced seller and one from the highest priced seller is

$$W_5 \equiv (1 + \delta)(V_1 + V_2) - 2p_L - p_H + \delta V_3$$

Proposition 1 *The buyer buys two units in period 1.*

Thus, we argue that the buyer will decide in equilibrium whether to split her order in period 1 or buy both units from the low priced seller. We make the argument in a series of Lemmas.

Lemma 4 *The sellers set strictly positive prices in period 1.*

P proof. If a seller set a price of 0, then either the buyer would buy two units from that seller or one from each of the sellers. In either case, the seller makes 0 profit. If the buyer buys two units, then the seller can sell no more in period 2. If he sells one unit, then the buyer must have bought one unit from the other seller, since $W_2 > W_3$ at a 0 price in period 1. The seller could raise his price to a price less than δV_3 and the buyer will buy at least one unit from the seller due to Lemma 9. ■

Lemma 5 *The buyer never buys three units in period 1.*

P proof. By Lemma 1, both prices are positive. Since $W_1 > W_5$ if $P_L > 0$, then buying two units always dominates buying three units. ■

Lemma 6 *The buyer prefers to buy one unit from each of the sellers instead of only one unit from the low price seller if $p_H \leq V_2 + \delta [E \min[p_1, p_2] + E p_2] \equiv p^C$. Thus the buyer will not buy any units from a seller charging $p_H > p^C$, when $p_H > p_L$.*

P proof. Compare W_1 and W_3 . ■

Lemma 7 *In any equilibrium, each price offered by each seller in period 1 is an offer which results in his selling at least one unit with positive probability.*

P proof. Let \bar{p} be the highest price offered in any (possibly mixed strategy) equilibrium by a seller. Suppose that in equilibrium \bar{p} is never accepted. By Lemma 3 the seller's expected payoff of making this offer is δV_3 . Let \underline{p} be the lowest price offered in equilibrium by the other seller. By Lemmas 6 and 9 if the difference between the two prices set is less than δV_3 , then the highest price will always be accepted, as long as it is less than p^C . By Lemma 1 $\underline{p} > 0$. A player offering \bar{p} can defect and offer a price p that is the minimum of $[\underline{p} + \delta V_3, p^C]$ and know that it will be accepted, and increase his payoff. Thus, no offer is made that is always rejected. ■

Thus, we will never be in a situation where the buyer accepts no goods in period 1, since the lowest of the two prices will always be accepted for at least 1 unit. So, at this point we know that the buyer will buy at least one unit, but not more than two units in period 1.

Lemma 8 *In any equilibrium, no seller will offer a price above p^C .*

P roof. Let \bar{p}_i be the highest price offered in any equilibrium by seller i . Suppose that $\bar{p}_i \geq \bar{p}_j$, $i \neq j$, and $\bar{p}_i > p^C$. If $\bar{p}_i > \bar{p}_j$, then seller i 's offer will never be accepted by Lemma 6. Then seller i 's payoff is δV_3 . Seller i can clearly improve his payoff by making an offer of $\delta V_3 + \epsilon$ (note that $\delta V_3 + \epsilon < p^C$). Suppose now that $\bar{p}_i = \bar{p}_j \equiv \bar{p}$. There could not be a mass point at \bar{p} by each seller, since only one unit will be bought and that seller could increase his payoff by a slight undercut in price. If there is no mass at \bar{p} , then there is no possibility that the offer will be accepted. But, this contradicts Lemma 7. ■

Lemmas 6 and 8 imply that in equilibrium there are always two units purchased in period 1. Thus, we have proved Proposition 1. Hence, all the buyer would have to compare are the payoffs of W_1 and W_2 .

Lemma 9 *The buyer prefers to buy one unit from each seller as opposed to buying two units from the lowest priced seller, $W_1 \geq W_2$, if*

$$p_L + \delta V_3 > p_H$$

P roof. Compare W_1 and W_2 . ■

This is an important result. It says that a buyer prefers to split his order if the discounted price differential is lower than the discounted price of a third unit when facing a monopolist. The price of a third unit when splitting an order is zero, while if the buyer does not split an order it is V_3 .

Now, we argue that there is no pure strategy equilibrium (symmetric or asymmetric) in the entire game.

Proposition 2 *There is no pure strategy equilibrium in the monopsony, linear pricing model.*

P roof. Suppose we have a pure strategy equilibrium with prices p_H and p_L , where $p_H \geq p_L$. Let us call the seller charging price p_H , the high price seller, firm H and the seller charging price p_L , the low price seller, seller L . A pure strategy equilibrium could exist only if both sellers offered p^C and the buyer was purchasing a unit from each seller. At any other price, at least one of the sellers could defect and improve their payoff. To see this, we need to look at various cases. First, suppose that the lower offer in equilibrium, p_L , is greater than δV_3 . If $p_L > p_H + \delta V_3$, then the buyer will split his order by Lemma 9. Seller L could improve his profit by increasing his offer. If $p_L < p_H + \delta V_3$, then the buyer will buy both units from seller L . Seller H will have a payoff of δV_3 . Seller H can improve his payoff by making an offer that is accepted. If $p_L = p_H + \delta V_3$, then

either seller L is not selling 2 units or seller H is not selling any units. One of the sellers has an incentive to defect. To see this, suppose that $\alpha \in [0, 1]$ is the probability that the buyer splits his order between the sellers. Then the payoff to seller L is $\pi_L = \alpha p_L + (1 - \alpha)2p_L$. The payoff to seller H is $\pi_H = \alpha p_H + (1 - \alpha)\delta V_3$, which equals $\pi_H = \alpha p_L + \delta V_3$ by assumption that $p_L = p_H + \delta V_3$. But seller H 's payoff must be at least as large as $p_L + \delta V_3 - \epsilon$ for all positive ϵ , since he could always guarantee an acceptance by dropping his price ϵ . Thus, α would have to equal 1. But, if $\alpha = 1$, then seller L could improve his payoff by raising his price.

Now, suppose that $p_L \leq \delta V_3$. If $p_L > p_H + \delta V_3$, then the buyer will split his order by Lemma 9. Seller L could improve his profit by increasing his offer. If $p_L < p_H + \delta V_3$, then the buyer will buy both units from seller L . If $\frac{\delta V_3}{2} < p_L$, then seller H can improve his payoff by making an offer that is accepted. If $p_L < \frac{\delta V_3}{2}$, then seller L could raise his offer to p_H the buyer will split his order and the low seller's profit increases. As before, if $p_L = p_H + \delta V_3$, then one of the sellers could do better by defecting.

An equilibrium with both sellers offer p^C could arise only if $p^C \geq 2(p^C - \delta V_3)$ or if $2\delta V_3 \geq p^C$. This is because a defection by a seller that gets the buyer to buy two units from the seller will reduce his profits. Otherwise a seller would defect. This is equivalent to $2\delta V_3 \geq V_2 + \delta [E \min[p_1, p_2] + E p_2]$. But, this condition never can hold, since both p_1 and p_2 are greater than $V_3/2$. ■

The result of no pure strategy equilibrium is due to two phenomena. First, as depicted in Lemma 9, the buyer's incentive to split her orders if the prices are close, within δV_3 . This gives the sellers incentives to raise price. On the other hand, if prices get "high", then sellers have incentives to drop their prices, and sell two units immediately.

Thus far, we have proved there is no pure-strategy equilibrium in the game. Now let us further characterize the (mixed-strategy) symmetric equilibria of the game.

Proposition 3 $\bar{p} - \underline{p} \geq 2\delta V_3$

P proof. We know that $\bar{p} - \underline{p} \geq \delta V_3$; otherwise a seller could increase his payoff by moving mass from lower parts of the price distribution to higher parts and still get accepted. Suppose that $\bar{p} - \underline{p} < 2\delta V_3$ and for now assume that the equilibrium price distribution is continuous. Define three regions as follows: region 1 where $p \in [\underline{p}, \bar{p} - \delta V_3]$, region 2 where $p \in [\bar{p} - \delta V_3, \underline{p} + \delta V_3]$ and region 3 where $p \in [\underline{p} + \delta V_3, \bar{p}]$. A price offered in region 1 will be accepted for either 1 or for 2 units. A price in region 2 will always be accepted for 1 unit. A price in region 3 will be accepted either for

a single unit or no units. But, if there is an offer in region 2, then a seller can always improve his payoff by moving all the probability mass in region 2 to a price of $\underline{p} + \delta V_3$. Thus, there would be a gap in the price offer distribution.

Suppose that there was a gap in the price offer distribution in region 2. Then prices offered in region 1 would all be moved to the top of region 1 at a price of $\bar{p} - \delta V_3$, since whether the offer is accepted either once or twice is independent of the price in region 1. But, if sellers move up all their mass to $\underline{p} + \delta V_3$, then the price distribution would only be δV_3 , but then any price in the interior distribution is inferior to either a price at the bottom or the top of the distribution. Thus, we would have a two-point distribution. But this cannot be an equilibrium. Suppose one player made an offer of \underline{p} and the other at \bar{p} . Then either the buyer accepts 2 at the low price or splits his order. In the former case, the high bidder could improve his payoff by reducing his offer slightly, while in the latter case the low bidder could improve his payoff by a bid reduction. ■

This implies that there are three regions regarding acceptance, with the second region possibly being a single point.

[Insert Figure: prices between \underline{p} and $\underline{p} + \delta V_3$ either one or two acceptances, prices between $\underline{p} + \delta V_3$ and $\bar{p} - \delta V_3$, either 0, 1 or 2 acceptances. and price between $\bar{p} - \delta V_3$ and \bar{p} either 1 or 0 acceptances].

Let π be the equilibrium payoff. The equilibrium pricing equations are

If $p < \underline{p} + \delta V_3$

$$p[2 - F(p + \delta V_3)] = \pi \quad (7)$$

If $\underline{p} + \delta V_3 < p < \bar{p} - \delta V_3$

$$\delta V_3 F(p - \delta V_3) + P[2 - F(p + \delta V_3) - F(p - \delta V_3)] = \pi \quad (8)$$

If $p > \bar{p} - \delta V_3$

$$\delta V_3 F(p - \delta V_3) + P(1 - F(p - \delta V_3)) = \pi \quad (9)$$

Finally, let us note the following important equilibrium facts.

Lemma 10 *The lowest price offered in equilibrium, \underline{p} , is greater than δV_3 .*

P proof. Substituting $p = \underline{p}$ and $p = \underline{p} + \delta V_3$ into (7), setting the two resulting values equal to

each and manipulating the equation, we obtain

$$\delta V_3 \left[2 - F(\underline{p} + 2\delta V_3) \right] = \underline{p} [F(p + 2\delta V_3) - F(p + \delta V_3)].$$

Since $2 - F(\underline{p} + 2\delta V_3) \geq 1$ and $F(\underline{p} + 2\delta V_3) - F(\underline{p} + \delta V_3) < 1$, it must be the case that $\underline{p} > \delta V_3$. ■

An immediate corollary of Lemma 10 is:

Corollary 1 *In the monopsony model with linear prices the expected profit of each seller is greater than δV_3 .*

Thus, the sellers receive rents above satisfying the residual demand after the buyer bought the other seller's capacity. Need strategic uncertainty for the sellers to make an offer with probability 1 in equilibrium. By not having any uncertainty about what a buyer will do given the prices offered by sellers, sellers can increase their expected payoff over what it would be in standard static Bertrand competition. Thus, sellers have expected economic rents above the profit that they receive from satisfying the buyer's residual demand curve by the other seller's capacity.

From Corollary 1 we obtain two additional Corollaries:

Corollary 2 *In the monopsony model with linear prices, the buyer would like to commit to not buying any units in period 2, unless the discount factor, δ , is too low.*

In other words, the buyer would be better off to commit to make all his purchases in a single period. To see this point, note that we have shown in Lemma 3 that the second period equilibrium if no units are sold in period 1 has each seller making an expected profit of V_3 . Thus, V_3 would, of course, also be the sellers' expected payoff if all competition took place in one period (as a result of the buyer's commitment to not make any purchases in the future), since the strategic situation would be exactly the same. Since the allocation is always efficient, lower seller profit implies higher buyer profit. In the equilibrium of the game with purchases (potentially) over two periods, the expected buyer's payment (to both sellers) is strictly above $2\delta V_3$ and, thus, strictly exceeds his expected payment of $2V_3$, if δ is not too low.

The result that a buyer may have an incentive to commit to make all the purchases in one period is interesting. For instance, this is consistent with the practice of airlines placing a large order that often involves the option to purchase some planes in the future. Consistent with our analysis, such orders come with a payment today (and commitment, even though not in full) for

units for which there is no need until future periods. Such behavior is sometimes attributed to economies of scale - our analysis shows that such behavior may emerge for reasons purely having to do with how sellers compete with one another.¹²

There is a second Corollary, also following from Corollary 1:

Corollary 3 *In the monopsony model with linear prices, the buyer has a strict incentive to buy one of the sellers, that is, to become vertically integrated.*

This result is based on the following calculations. By vertically integrating, and paying the equilibrium profit of a seller, π , the total price that the buyer will pay is $\pi + \delta V_3$ since the other buyer would change the monopoly price V_3 for a third unit (sold in period 2). This total payment is strictly less than his total expected payment in equilibrium of 2π . Thus, even though the other seller will be a monopolist, the buyer's payments are lower, since the seller that has not participated in the vertical integration now has lower profits.

4 Monopsony with non-linear pricing

Suppose that a firm can offer a menu of prices in each period; a price if it sells one good and a price that it sells two goods.

4.1 Second period

First, we consider equilibrium in the possible period 2 subgames.

Buyer bought two units in period 1. In this case, non-linear pricing is the same as linear pricing (since at most one unit can be bought in period 2). If the buyer bought a unit from each seller in period 1, the price that each seller charges in period 2 for a single unit is 0. If the buyer bought two units from one of the sellers, then the price for a unit of the other seller is V_3 . Again, the period 2

¹²It is also easy to see that the buyer would be better off if he could commit to reduce his demand to only two units. By committing to not purchase a third unit, the value he obtains gets reduced by δV_3 , while his payment gets reduced by more (since the sellers' equilibrium profit drops from $\pi > \delta V_3$ to zero). Still, it should be noted that commitment to such behavior may be difficult: once the initial purchases have been made, the buyer would then have a strict incentive to "remember" his demand for a third unit. Similarly, the sellers would have an incentive not to reveal some of their available capacity, as such a strategic move (if credible) would lead to higher profit for them. Similar remarks, as the ones made just above, regarding the (non) credibility of such strategies hold.

equilibrium profit of a seller that has one remaining unit of capacity is 0 and that of a seller with two remaining units is V_3 .

Buyer bought one unit in period 1. The equilibrium price triples satisfy $\{p_1, p_2^1, p_2^2\} = \{0, k, V_3\}$, where $k \geq V_3$. The firm that has one unit of capacity (seller 1) charges $p_1 = 0$ for his unit and the firm with two units of capacity (seller 2) charges V_3 if the buyer buys two units (p_2^2) from the seller and a price that is not lower than that if the buyer buys one unit (p_2^1). In equilibrium, the buyer buys both units from seller 2.

First, we check to see that the above strategy profiles are an equilibrium, next we argue that these are the unique pure strategy equilibrium payoffs. The buyer is indifferent between buying both units from seller 2 or one unit from seller 1: either way, his net surplus is equal to V_2 . If seller 1 raises his price, he will still sell no units and his profit remains at zero; lowering his price would result in a loss. If seller 2 raises p_2^2 he will sell nothing and his profit drops to zero (to be compared to the profit of V_3 at the candidate equilibrium). If seller 2 lowers either price below V_3 , then that price will be accepted and the seller will have a payoff lower than V_3 . Thus, this is an equilibrium.

Now, we argue that the equilibrium payoffs are unique. First, in any equilibrium the buyer will buy two units: if we had a candidate equilibrium where the buyer bought only one unit (from either seller 1 or seller 2) then seller 2 could charge an incremental price of less than V_3 and sell his second unit, thus increasing his profit. Second, there cannot be an equilibrium where the buyer gets one unit from each seller. For the buyer to buy one unit from each seller, it must be that $p_1 + p_2^1 \leq p_2^2$ and each price must not exceed V_3 . Then seller 2, has a profitable deviation to setting p_2^2 equal to the original p_2^1 plus $p_1/2$ and setting the price for one period so that the buyer will never buy only one unit from him. Third, we must have $p_2^2 = p_1 + V_3$. Otherwise, a seller could defect. But then seller 1 could do better by lowering his price by ϵ and make a sell if $p > 0$. Thus, we have unique equilibrium payoffs.

Buyer bought no units in period 1. The prices would be each seller charging V_3 whether the buyer buys one or two units and the buyer buys two units from one seller and one from the other. The buyer's net surplus is $V_1 + V_2 + V_3 - 2V_3$ and each seller's profit is V_3 . Let us establish that this is an equilibrium. If a seller increased his price he would sell no units and his profit would drop (from V_3 to zero). If a seller decreased either of the prices he charges (that is, the price for one or for two units), this price would get accepted and the seller's payoff would drop from V_3 to the new price level.

The equilibrium is unique. To see this, first note that we cannot have an equilibrium where the buyer buys fewer than 3 units. This is because, if that were the case, one seller would have a strict incentive to lower one of his prices. Furthermore, a seller can always guarantee himself a profit of V_3 since the buyer is always willing to pay this amount for the third unit. Finally, using simple arguments we can show that Bertrand competition will induce both sellers to price using two part-tariffs with the fixed portion of V_3 and marginal costs of 0.

Equilibrium behavior in period 2, under non-linear pricing, can be summarized as follows.

Lemma 11 *Second period competition in the case of a monopsonist under non-linear pricing falls into one of three categories. (i) If only one seller is active (the rival has zero remaining capacity), that seller sets the monopoly price, V_3 , and extracts the entire seller's surplus. (ii) If both sellers have enough capacity to cover the buyer's demand, there is (Bertrand) pricing at zero. (iii) If the buyer's demand exceeds the capacity of one seller but not the aggregate sellers' capacity then there is a pure strategy equilibrium: a seller with two units charges V_3 for two units (and a price at least as high for one unit) and a seller with one unit charges 0.*

It follows from the above analysis that a seller gives up second-period profit V_3 when, by selling one (or two) units in period 1, his remaining capacity drops from 2 units to 1 (or 0, respectively). Clearly, he would demand at least the discounted present value of that amount to sell one unit in period one.

Discussion. Comparing competition under linear and non-linear pricing, we observe that there is a critical difference in the case when the buyer's demand exceeds the capacity of one seller but not the aggregate sellers' capacity. Under linear pricing, a pure strategy equilibrium fails to exist because the seller with two units of capacity cannot prevent the price of the first unit affecting his sales of a second unit, while he can achieve this under non-linear pricing. To see why, suppose that under linear pricing, the seller with two units of capacity, seller 2, charges V_3 . The seller with one unit of capacity, seller 1, would respond by charging $V_3 - \epsilon$ to guarantee a sell. But, seller 2's best reply would be to slightly undercut seller 1 and sell both units. This undercutting process would take place until both prices reached $V_3/2$ because, at that point, seller 2 would prefer to sell only one unit at a price of V_3 . On the other hand, with non-linear prices, seller 2 can implicitly keep the undercutting process going by bundling his two units. This, essentially, puts the price of the first unit that each seller has to a zero price, while maintaining the price for seller 2's second unit at his monopoly price of V_3 . To guarantee that the buyer buys seller 2's bundle, seller 2 raises the

price of buying only a single unit to at least V_3 .

4.2 First period

Now we go back to period 1. There is a unique pure strategy equilibrium price paid for the goods. The equilibrium prices are for each seller to charge δV_3 for both a single unit and two units. The buyer either buys one good from each seller, two units from one of the sellers and none from the other, or two units from one seller and one unit from the other. The total amount paid by the buyer and the revenue that each seller receives is the same, no matter which of the three actions that the buyer takes in period 1.

Given the equilibrium prices, if the buyer accepted one unit from each seller, his expected payoff over the two periods would be

$$\tilde{W}_1 \equiv (1 + \delta)(V_1 + V_2) - 2\delta V_3 + \delta V_3.$$

If the buyer accepted *two* units from one of the sellers his payoff would be

$$\tilde{W}_2 \equiv (1 + \delta)(V_1 + V_2) - \delta V_3.$$

Now, we argue that the strategy profile described above is an equilibrium. If a seller raised his price for a single unit, then the buyer would buy both units from the other seller in period 1. The seller then becomes the only one with available capacity in period 2 and hence his overall payoff over the two periods is δV_3 . Thus, there is no improvement in the seller's payoff. Increasing the price for two units to some level \tilde{p} , has no effect on the buyer's choice, since the buyer prefers buying one unit from each seller and obtaining payoff \tilde{W}_1 to

$$\tilde{W}_3 \equiv (1 + \delta)(V_1 + V_2) - \tilde{p}$$

whenever $\tilde{p} > \delta V_3$. Thus, such a price increase cannot improve the seller's payoff. Clearly, lowering the price cannot improve a seller's payoff. Thus, this is an equilibrium.

Why are the equilibrium prices unique? Let us take a candidate equilibrium where each seller demanded a different price for a single unit. It is easy to show that both prices are at least δV_3 . Then either the lower price seller could increase his price, if the buyer split his order, or the higher priced seller who got no orders could reduce his price and make a sell. So, each seller has to offer the same price for a single unit.

Suppose now that each seller demanded a price $p_1 > \delta V_3$ for a single unit and $p_2 \geq p_1$ for two units. If the buyer buys one unit from each seller, his payoff over both periods is

$$(1 + \delta)(V_1 + V_2) - 2p_1 + \delta V_3,$$

since second period competition implies he will then get the third unit at zero price. If the buyer buys two units from the same seller, his payoff is

$$(1 + \delta)(V_1 + V_2) - p_2,$$

since the seller with remaining capacity will be a monopolist in period 2 and charge V_3 . The buyer's payoff is higher if he buys one unit from each seller if

$$p_2 > 2p_1 - \delta V_3$$

and he will buy two units from one of the sellers otherwise.

If $p_2 < 2p_1 - \delta V_3$, the buyer would accept both units from one of the sellers. The other seller would then have a payoff of δV_3 . He could improve his payoff by lowering his price for two units and having the buyer accepting both his units, since $p_2 \geq p_1 > \delta V_3$. If $p_2 > 2p_1 + \delta V_3$, the buyer would split his order. A seller could improve his payoff by lowering the price for two units to some price less than $2p_1 - \delta V_3$, the buyer will accept both his units and the seller will improve his payoff.

Another way to argue uniqueness for a block price of $p = \delta V_3$ is as follows. Suppose a seller is offering a distribution of block prices $F(\cdot)$. The profit of the other seller is

$$\pi(p) = p[1 - F(p)] + \delta V_3 F(p).$$

Taking the first order condition, we find

$$F(p) = 1 + f(p)[\delta V_3 - p].$$

Since F must be increasing in p , the highest point of the distribution is δV_3 . But, a seller would prefer to be a monopolist next period to making a lower offer that would be accepted.

We summarize as follows:

Proposition 4 *With a monopsonist under non-linear pricing, there are unique pure strategy equilibrium payoffs. In period 1, both sellers charge δV_3 for both a single unit and two units and the buyer buys either two or three units. Period 2 equilibrium is as stated in Lemma 11.*

The equilibrium involves a two-part tariff with the fixed fee equal to a seller's discounted monopoly profit in the next period and all units are priced at marginal cost, which we have normalized to 0. This equilibrium was not possible with linear prices, because each seller would have an incentive to raise his price, but not more than δV_3 which would induce the buyer to split his order.

Remark. Unlike the case when linear prices are used, under non-linear prices the sellers make no positive rents: their equilibrium payoffs δV_3 are equal to the profit from satisfying the residual demand, after the buyer bought the other seller's capacity. Further, the buyer has no incentive to commit to not making purchases in period 2, and has no incentive to vertically integrate, by buying one of the sellers, since sellers only receive the monopoly profits from satisfying the residual demand.

5 Duopsony

To study strategic issues raised when there are more than one buyers, we start by the case of two buyers ($N = 2$). Each buyer has the same demand as in the monopsony case, and each seller has now doubled his capacity to four units. Now, a buyer coordination issue which was not present in the monopsony case is introduced.

5.1 Linear prices

First, we study linear prices. Then we examine non-linear prices.

5.2 Second period

We consider equilibrium behavior in the possible period 2 subgames. All the proofs are similar to that of Lemma 2 and, thus, the details are omitted.

Each buyer bought two units in period 1. If both sellers sold two units in period 1, then the equilibrium has both sellers charging 0 in period 2. If one seller sold 4 units in period 1, then the equilibrium is for the other seller to charge V_3 and for each buyer to buy a unit. If one of the sellers sold 3 units in period 1, and the other sold 1 in period 1, then there is a unique mixed strategy equilibrium. Let seller 1 be the seller who has 1 unit of capacity in period 2 (that is, sold 3 units in period 1) and seller 3 be the one who has three units of capacity in period 2 (that is, sold 1 unit in period 1). The security profits, which are the unique equilibrium profits for seller 1 and 3, are

$V_3/2$ and V_3 . Denoting by F_1 and F_3 the distribution functions employed at the mixed strategy equilibrium by the two firms, satisfy conditions

$$p_1 [1 - F_3(p_1)] = \frac{V_3}{2}$$

and

$$p_3 [2(1 - F_1(p_1)) + F_1(p_3)] = V_3$$

where the prices are from $\left[\frac{V_3}{2}, V_3\right]$. It follows that seller 1's distribution is $F_1(p_1) = 2 - \frac{V_3}{p_1}$. As in the monopsony case of Lemma 2, seller 3's distribution will have a mass point at V_3 : it satisfies $F_3(p_3) = 1 - \frac{V_3}{2p_3}$, with a mass point of $1/2$ at V_3 .

One seller sold 2 units in period 1 and the other sold 1 unit in period 1, with each buyer buying at least one unit. Let seller 2 be the seller that has 2 units of capacity remaining and seller 3 have 3 units of capacity. The equilibrium profits are $2V_3/3$ for seller 2 and V_3 for seller 3. Now, the equilibrium pricing equations satisfy

$$2p_2 [1 - F_3(p_2)] = \frac{2V_3}{3}$$

and

$$p_3 [3(1 - F_2(p_3)) + F_2(p_3)] = V_3$$

where the prices are from $\left[\frac{V_3}{3}, V_3\right]$ and are distributed according to $F_2(p_2) = \frac{3}{2} - \frac{V_3}{2p_2}$ and $F_3(p_3) = 1 - \frac{V_3}{3p_3}$, with a mass point of $1/3$ at V_3 .

Each seller sold 1 unit in period 1 and each buyer bought one unit. Then the equilibrium prices would be distributed on $\left[\frac{V_3}{3}, V_3\right]$ and satisfy

$$p [3(1 - F(p)) + F(p)] = V_3$$

or,

$$F(p) = \frac{3}{2} - \frac{V_3}{2p}.$$

with each seller having an expected payoff of V_3 .

One seller sold two units and the other none, and either each of the two buyers bought one unit or one buyer bought two units. The equilibrium payoff of the seller who sold no units is $2V_3$, while the equilibrium payoff of the other seller is V_3 .

One seller sold 1 unit and the other none. Suppose that Seller 3 sold 1 unit in period 1, he has 3 units of capacity in period 2, and seller 4 sold none in period 1, he has 4 units of capacity in period

2. Then the equilibrium prices are distributed on $\left[\frac{V_3}{2}, V_3\right]$ and the corresponding distributions satisfy

$$p_3 [3(1 - F_4(p_3)) + F_4(p_3)] = 3V_3/2$$

and

$$p_4 [4(1 - F_3(p_4)) + 2F_3(p_4)] = 2V_3.$$

Seller A 's price distribution is $F_3(p_3) = 2 - \frac{V_3}{p_3}$ and seller A 's distribution is $F_4(p_4) = \frac{3}{2} - \frac{3V_3}{4p_4}$ with a mass point of $1/4$ at V_3 for F_4 . The equilibrium payoffs are $3V_3/2$ for seller 3 and $2V_3$ for seller 4.

Neither seller sold a unit in period 1. Then the equilibrium price distribution for each seller satisfies

$$p_4 [4(1 - F(p_4)) + 2F(p_4)] = 2V_3$$

or,

$$F(p_4) = 2 - \frac{V_3}{p_4}$$

on $\left[\frac{V_3}{2}, V_3\right]$. The equilibrium expected payoffs are $2V_3$.

The key results for period 2 are summarized in the following Lemma.

Lemma 12 *In the second period of a duopsony under linear pricing (i) the highest expected payoff for a seller with full capacity is $2V_3$; (ii) If one of the buyers bought 2 units in period 1 and the other buyer bought none, then the expected price is more than $V_3/2$ per unit in period 2; (iii) If each buyer bought 2 units in period 1, with one of the sellers selling 3 units and the other 1 unit, then the price in period 2 is more than $V_3/2$.*

5.3 First period

Now, we go to period 1. As in the monopsony case, there will be no pure strategy equilibrium, but the equilibrium behavior will have a very different flavor. We focus on the following equilibrium. Each seller asks δV_3 for each unit and the buyers mix with equal probability between buying two units from either seller. First, we argue why this is an equilibrium and then discuss the properties of the equilibrium. We then discuss another equilibrium.

Suppose that one seller (say seller L) is charging p_L per unit and the other seller (seller H) is charging p_H . Suppose that the buyers mix between buying two units from seller L with probability α and buying two units from seller H with probability $(1 - \alpha)$. The payoff of a buyer buying from

seller L is

$$(V_1 + V_2)(1 + \delta) - 2p_L + (1 - \alpha)\delta V_3.$$

With probability α , the other buyer also buys from seller L , in which case only seller H will have available capacity in period 2 and, acting as a monopolist will leave the consumers with zero surplus. With probability $1 - \alpha$, the other buyer buys from seller H , in which case there is Bertrand competition between the two sellers in period 2, leaving surplus V_3 to each of the buyers. Similarly, the payoff of a buyer buying from seller H can be calculated to be

$$(V_1 + V_2)(1 + \delta) - 2p_H + \alpha\delta V_3.$$

Thus, for the buyer to be indifferent between buying from seller L and H we must have

$$\alpha = \frac{p_H - p_L}{\delta V_3} + \frac{1}{2}.$$

Note, that at $p_H = p_L = \delta V_3$ buyers mix with probability $1/2$.

Let $f(p_L)$ be the density of prices offered by seller L which range in some interval \underline{p}_L to \bar{p}_L . Seller H 's expected payoff over the two periods is

$$\pi_H(p_H) = 2\delta V_3 \int_{\underline{p}_L}^{\bar{p}_L} \alpha^2 f(p_L) dp_L + 2p_H \int_{\underline{p}_L}^{\bar{p}_L} 2\alpha(1 - \alpha) f(p_L) dp_L + 4p_H \int_{\underline{p}_L}^{\bar{p}_L} (1 - \alpha)^2 f(p_L) dp_L$$

which simplifies to

$$\pi_H(p_H) = 2\delta V_3 \int_{\underline{p}_L}^{\bar{p}_L} \alpha^2 f(p_L) dp_L + 4p_H \int_{\underline{p}_L}^{\bar{p}_L} (1 - \alpha) f(p_L) dp_L.$$

Differentiating with respect to p_H , we find $p_H = \delta V_3$ for any density $f(p)$. This is a local maximum, since π_H is strictly concave in p_H . The calculations for seller L 's payoff are similar. Furthermore, it is easy to show that no deviation by a seller that makes the probability of acceptance either 0 or 1 will improve his payoff, which can be calculated in equilibrium to be equal to $\frac{5\delta V_3}{2}$. Thus, it is a unique best response for the sellers to each ask δV_3 , given the buyers' purchasing strategies.

We need to make sure that each buyer is acting optimally, given the strategies of the other players. Suppose that buyer 1 is following the putative equilibrium strategy. The payoff for buyer 2 from following the putative equilibrium strategy (that is, to not split his order) is

$$S_{ns} = (V_1 + V_2)(1 + \delta) - 2\delta V_3 + 1/2\delta V_3. \quad (10)$$

If buyer 2 splits his order, by buying one unit from each seller, this his payoff is

$$S_s = (V_1 + V_2)(1 + \delta) - 2\delta V_3 + \delta(V_3 - Ep_2)$$

where Ep_2 is the expected price that the buyer will have to pay in period 2, given that one seller has sold three units and the other one unit in period 1. Payoff S_{ns} is larger than S_s , since $Ep_2 > \frac{\delta V_3}{2}$ by Lemma 12. It is also easy to show that the buyer prefers to buy two units in period 1 to any other quantity. Thus, we have an equilibrium.

It is important to note that it would not be an equilibrium if the buyers coordinated their behavior by either buying from different sellers with probability 1 or by splitting their orders, buying one unit each from each seller. This would be in the buyers' best interests, given the sellers' prices, since then they would pay $2\delta V_3$ in period 1 and nothing for the third unit in period 2. We now argue why this coordination by buyers cannot be part of an equilibrium. Suppose that buyers did coordinate their behavior when the sellers each asked δV_3 per unit. A seller could raise his price by ϵ and improve their payoff. There are two possible responses by buyers to this deviation. They could make the same purchasing decisions as before, either splitting or each buying from one seller with probability one. In either case, this improves the deviator's payoff. Or, the buyers could mix their purchasing decisions.¹³ In this case, the seller would sell either 0, 1, 2, 3, or 4 units. If he sells a positive number of units in period 1 he improves his payoff, since the payoff is at least $2\delta V_3 + \epsilon$. This is because if a seller sell only one unit his payoff is $\delta V_3 + \epsilon$ in period 1 and δV_3 in period 2. If he sells no units, then his payoff is $2\delta V_3$. Since, the buyers must be mixing, then the seller's expected sells are positive, thus he improves his payoff. It should be noted that the buyers will each buy two units in period 1. Further, it is easy to show that if the sellers offered the same price, but one that was different than δV_3 , than at least one of the sellers could improve their payoff by defecting.

Proposition 5 *In the linear pricing duopsony game there is a symmetric equilibrium. In period 1, each seller demands δV_3 per unit and the buyers mix equally between buying two units from one or the other seller. In period 2, the prices are 0 per unit if the buyer bought from different sellers and V_3 if they bought from the same seller.*

Properties of the equilibrium. The sellers get more than the profits that they would get by not selling any units in period 1, and then becoming a monopolist in period 2. Such a strategy would generate a profit of $2\delta V_3$, which is less than their equilibrium profit of $\frac{5\delta V_3}{2}$. This is because the buyers are mixing their purchasing decisions between the sellers. By mixing there is the possibility that the sellers sell 0 or 4 units with probability 1/4 and selling 2 units with probability 1/2. This

¹³Note that it is never an equilibrium where both buyers would buy from the same seller with probability one, since a buyer would do better by accepting the other seller's offer.

is desirable from the sellers' points of view, since if they do not make a sell in period 1, then they become a monopolist in period 2 and receive a payoff of $2\delta V_3$. Thus, the sellers like that the buyers do not coordinate their behavior.

Thus, we need strategic uncertainty from the sellers' points of view to get them to charge δV_3 . In contrast, it was not possible to get strategic uncertainty when there was only a single buyer. This is because of two reasons. First, the single buyer did not have anyone to "miscoordinate" with on purchases. Second, if the buyer mixed, then a seller could improve his payoff by changing his price. Thus, sellers must mix to get an equilibrium. This allows them to obtain an equilibrium payoff above $2\delta V_3$.

From the above conclusion (about the sellers' expected profit in equilibrium), we obtain two Corollaries, similar to these stated in the monopsony case.

Corollary 4 *The buyers have a strict incentive to commit not to buy in the future, unless the discount factor, δ , is too low.*

If both buyers could commit in period 1 not to buy in period 2, then they would both be better off. Our analysis above has shown that each seller's equilibrium profit and thus each buyer's equilibrium payment if no units have been sold in period one is $2V_3$. This amount, which is the equilibrium payment of a seller if there is commitment to make all purchases in a single period, has to be compared to the equilibrium payment, $\frac{5\delta V_3}{2}$, when purchases can be made (potentially) over both periods. Clearly, the former amount is lower than the latter, as long as δ is not too low.

Also, like in the monopsony case, we have:

Corollary 5 *A buyer has a strict incentive to vertically integrate with a seller.*

Suppose that a buyer unilaterally buys a seller. This increases his expected profit because a buyer can buy a seller, by paying the equilibrium profit of $\frac{5\delta V_3}{2}$. This is the buyer's expected cost. He can then satisfy his demand for 3 units and have one extra unit left that he can supply to the other buyer and obtain an additional positive profit.

One may wonder if there is an asymmetric pure strategy equilibrium where each buyer buys two units from a different seller. We now argue why this is not the case. First, observe that both prices must be at least δV_3 ; otherwise a seller could do better by charging an infinite price in period

1 and selling two units at V_3 in period 2. The price of the low priced seller must satisfy

$$4(p_H - \delta V_3/2) \leq 2p_L; \quad (11)$$

otherwise, the low priced seller could deviate and get both buyers to buy from him. Clearly, to satisfy (11) one would want to minimize the difference between p_H and p_L . But, substituting $p_L = p_H$ into (11) we find that p_H must be less than δV_3 . Since, there is no equilibrium where prices are smaller than δV_3 , there can be no asymmetric pure strategy equilibrium.

There is another equilibrium where both sellers set the same price and the buyers mix between buying two units from each seller or splitting their orders and getting one unit from each. We demonstrate the equilibrium in the Appendix. It gives the sellers a lower profit than the one that we just derived, but still gives them a profit greater than $2\delta V_3$.

5.4 Non-linear pricing

We now sketch the results when the sellers can use non-linear prices.

5.4.1 Second period

Suppose that the sellers can offer non-linear pricing. If the buyers each bought two units in period 1, then the analysis is the same as when pricing is linear in period 2, since each buyer only wants a single unit. If each seller sold two units, then the price at which they sell additional units in period 2 is 0. If one seller sold four units, then the other seller will charge V_3 for a unit. If one seller sold three units and the other one unit, then there is a mixed strategy equilibrium. The seller who sold three units (has one unit of capacity) would have a price distribution of $F_1(P_1) = 2 - \frac{V_3}{P_1}$, while the seller who sold a single unit (has three units of capacity) would have a price distribution $F_3(P_3) = 1 - \frac{V_3}{2P_3}$, with a mass point of $1/2$ at V_3 . Both are distributed on $[\frac{V_3}{2}, V_3]$. We will assume that the buyers always buy two units in period 1— this is what will happen in equilibrium.

5.4.2 First period

Now, let us go back to period 1. The following is an equilibrium of the game. There are two possible equilibrium strategies for sellers. Either sellers demand $2\delta V_3$ if a buyer wants 2 or fewer units and $2.5\delta V_3$ or more if a buyer wants 3 units. Or, the sellers charge δV_3 per unit for the first two units and $0.5\delta V_3$ or more for a third unit. The buyers mix equally between buying two units from either seller. We need to check that this is an equilibrium. First, let us look at the buyers.

If buyer 1 is mixing equally between buying from either seller 2 units, then we have already shown in the linear pricing case that the payoff for the buyer is higher than if the buyer split his order between the two sellers. If buyer 2 buys two units from a seller then her expected payoff is

$$(V_1 + V_2)(1 + \delta) + \delta V_3 - 0.5(2\delta V_3) - 0.5(2\delta V_3 + \delta V_3)$$

where the first negative term represents the buyers cost if the buyers buy from different sellers in period 1, while the second if they buy from the same seller. That is, the expected price for the third unit in period 2 is $0.5V_3$. Thus, the buyer is indifferent between buying and not buying a third unit in period 1. Thus, the buyers are behaving optimally.

Now, let us turn to the sellers. The sellers' expected payoffs are $2.5\delta V_3$. First, we note that, given the buyers' strategies, a seller's expected payment for the third unit is $0.5\delta V_3$. This is because with probability 0.5, both buyers buy a third unit for zero in period 2, since they bought from different sellers in period 1, with probability 0.25 the seller sold all his units in period 1, and with probability 0.25 he sold no units in period 1 and charges V_3 in period 2 and sells 2 units. We have already showed that buyers will not pay more than $0.5\delta V_3$ so if a seller raises his price he will not sell the good, but if he lowers his price a buyer will buy the good, but that will be at a price less than a seller's expected price.

Take the case when sellers are demanding the same price whether the buyer buys one or two units. Suppose that one seller (say seller L) is charging p_L for two units and that the other seller (seller H) is charging p_H for two units. If α is the probability that both buyers will buy two units from seller L , it can be shown, using arguments similar to those used in the linear case, that $\alpha = \frac{p_H - p_L}{2\delta V_3} + \frac{1}{2}$. Let $f(p_L)$ be the density of prices offered by seller L , which range in some interval p_L to \bar{p}_L . Seller H 's expected profit function over the two periods is

$$\pi_H(p_H) = 2\delta V_3 \int_{p_L}^{\bar{p}_L} \alpha^2 f(p_L) dp_L + 2p_H \int_{p_L}^{\bar{p}_L} \alpha(1 - \alpha) f(p_L) dp_L + 2p_H \int_{p_L}^{\bar{p}_L} (1 - \alpha)^2 f(p_L) dp_L,$$

or,

$$\pi_H(p_H) = 2\delta V_3 \int_{p_L}^{\bar{p}_L} \alpha^2 f(p_L) dp_L + 2p_H \int_{p_L}^{\bar{p}_L} (1 - \alpha) f(p_L) dp_L.$$

Taking the first order condition, it can be shown that the profit maximizing price for seller H is $p_H = 2\delta V_3$. Thus, we have an equilibrium and the equilibrium payoff is unique. It can be shown, using similar arguments, that charging δV_3 per unit for the first two units is also an equilibrium.

This equilibrium has a very similar structure to the linear pricing equilibrium. The only real difference is that the price for the third unit must be set so that the buyers are not willing to buy a third unit and the sellers do not want to lower the price.

6 Many buyers

Suppose now that there are N buyers, each identical to the buyers in our analysis thus far. Let us look at linear pricing; as in the duopsony case, the equilibria are essentially the same in the non-linear pricing model. We focus on the equilibria where buyers buy two units from the same seller in period 1. Let the sellers each have $2N$ units of capacity. If seller A sold $N_A \geq N$ and seller B sold $2N - N_A$ units in period 1, then security profits in period 2 are $\frac{(N_A - N)(2N - N_A)V_3}{N}$ and $(N_A - N)V_3$ for sellers A and B , respectively. The equilibrium price distributions would satisfy

$$p_A [(2N - N_A)(1 - F_B(p_A))] = \frac{(N_A - N)(2N - N_A)V_3}{N}$$

and

$$p_B [N(1 - F_A(p_B)) + (N_A - N)F_A(p_B)] = (N_A - N)V_3$$

or

$$F_A(p) = \frac{N}{2N - N_A} - \frac{(N_A - N)V_3}{(2N - N_A)p}$$

$$F_B(p) = 1 - \frac{V_3(N_A - N)}{pN}$$

The prices are distributed on $\left[\frac{V_3(N_A - N)}{N}, V_3\right]$ with B having a mass point of $\frac{N_A - N}{N}$ at V_3 . Notice that with N_A close to N , the equilibrium expected price converges to 0 and the equilibrium profit converges to 0.

In period 1, each seller demands δV_3 per unit. The buyers mix equally between buying from each of the sellers. The sellers' expected payoffs are greater than $N\delta V_3$. To see this, let's look at the realization of each possible set of purchases in period 1. A seller's expected payoff if $N/2$ or fewer buyers buy from him in period 1 is $N\delta V_3$. This is because this seller has revenue of $\delta V_3(2N - N_A)$ in period 1, and gets a payoff of $V_3(N_A - N)$ in period 2, since this is his security payoff. After discounting period 2 payoffs, this sums to $N\delta V_3$. The payoff of a seller who attracts more than $N/2$ buyers is $\left[N + \frac{(N_A - N)}{N}(3N - N_A)\right]\delta V_3$. This seller gets a payoff of $N_A\delta V_3$ in period 1. In period 2, the seller's payoff is $\frac{(N_A - N)(2N - N_A)V_3}{N}$. Thus, as in the model with either a single buyer or two buyers, the sellers make expected economic rents beyond just supplying the residual demand curve.

The distribution of acceptances follows a binomial formula in period 1. The probability of k buyers choosing a seller with N buyers, given that they pick each seller with probability 0.5 is

$$\frac{N!}{k!(N - k)!} 0.5^N$$

For N large, this is approximated by a normal distribution with mean $N/2$ and standard deviation $0.5N^{1/2}$.

It is well known that for N large and the probability of an event occurring close to 0.5, that the binomial is closely approximated by a normal distribution. Thus, a seller's expected profits are

$$\pi(N) = \left[N + \int_N^{2N} \left(\frac{(N_A - N)}{N} (3N - N_A) \right) \Sigma(N/2, (N/2)^{0.5} dN_A \right]$$

where $\Sigma(N/2, (N/2)^{0.5}$ denotes the normal distribution with mean $N/2$ and standard deviation $(N/2)^{0.5}$. This is clearly increasing in N and the profits are increasing at a rate faster than N . By adding a buyer, there is a mean preserving spread of the distribution and the seller's payoff function is convex in N - see Figure 3.

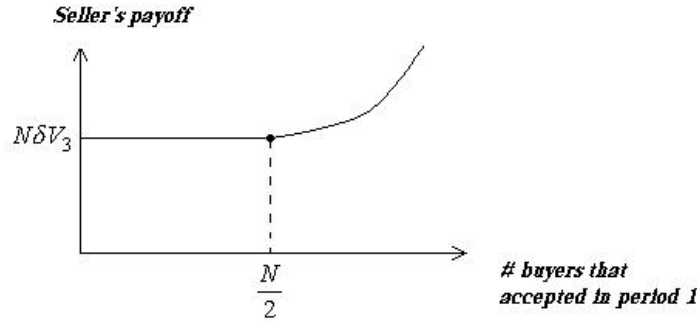


Figure 3: Equilibrium payoffs

7 Conclusion

In this paper, we have analyzed a market where both sellers and buyers act strategically. Sellers have intertemporal capacity constraints, as well as the power to set prices. Buyers decide which sellers to buy from, taking into consideration that their current purchasing decisions affect the intensity of sellers' competition in the future. We examine a set of simple dynamic models with two sellers; the models differ in whether there is a single or multiple buyers and in whether the sellers set linear or non-linear prices. In the monopsony case with linear prices, capacity constraints imply that a pure strategy equilibrium fails to exist. Instead, sellers play a mixed strategy with respect to their pricing, and the buyer may split his orders. The buyer has a strict incentive to commit not to buy in the future and to vertically integrate with one of the sellers. When non-linear pricing is feasible, however, a pure strategy equilibrium exists. In the case of two or more buyers, it is the buyers that must randomize their actions. As a result, equilibrium behavior does not need to

prescribe that buyers split their orders. As in the monopsony model, there are strict incentives for buyers to commit not to buy in the future or to vertically integrate. Finally, we observe that as the number of buyers grows, profits grow faster than linearly and the distribution of sales can be quite asymmetric.

While we have tried to keep the model as simple as possible, our qualitative results appear robust to modified formulations. The most important ones refer to how the capacity constraints function. In the model, if a seller sells one unit today his available capacity decreases tomorrow by exactly one unit. In some of the cases for which our analysis is relevant, like the ones mentioned in the Introduction, it may be that the capacity decreases by less than one unit, in particular, if we adopt the view that each unit takes time to build and, thus, occupies the firm's production capacity for a certain time interval. Similarly, instead of unit cost jumping to infinity once capacity is reached, in some cases it may be more realistic to assume that unit cost increases but in a smoother way: as known from similar cases, cost curves that are convex enough could play the role of capacity constraints. We believe the spirit of our main results is valid under such modifications, as long as the crucial property that by purchasing a unit from a seller you decrease this seller's supply in the subsequent periods holds.

This is, to our knowledge, the first paper that considers capacity constraints and buyers' strategic behavior in a dynamic setting - a number of extensions are open for future work. These present theoretical interest and, at the same time, may make the analysis more directly relevant for particular markets. First, it may be interesting to examine the case where the products offered by the two sellers are differentiated. If the buyers do not view the two products as homogeneous, a number of new issues arise. When would a buyer have an incentive to purchase a product other than his otherwise preferred one, simply with the purpose of intensifying competition in the future? Do buyers, in equilibrium, end up with purchasing different varieties? When there are multiple buyers, do the buyers end up purchasing over time from the same seller, thus forming what would appear to be an exclusive relationship? If there is a distortion because buyers strategically purchase products different from their most preferred ones?

A second interesting extension is when the sellers have asymmetric initial capacities. How different capacities affect pricing? Which seller sells faster? Do buyers have an incentive to favor a seller with a larger or with a smaller remaining capacity? Is competition more intense when capacities are more or less symmetric?

A third extension, based on the one described just above, is that of endogenizing the sellers'

capacities. Previous work has examined this issue as a two-stage game, where first capacities are chosen and then firms are competing for the final demand. However, regardless of whether final-stage competition is in quantities (e.g. Kreps and Scheinkman, 1983) or in prices (e.g. Allen, Deneckere, Faith and Kovenock, 2000) the issue of strategic buyers has not been treated. An important aspect of an analysis in this case would be how strategic behavior from the buyers' side affects the sellers' incentives to build additional capacity.

Price determination takes in our model a relatively simple form with sellers setting prices (linear or non-linear) in each period. A fourth possible extension would be to examine alternative pricing formulations. For instance, sellers may be able to make their prices dependent on the buyers' purchasing behavior e.g. by offering a lower price to a buyer that has not purchased in the past (or does not currently purchase) a unit from the rival seller. Our setting may allow us to examine such "loyalty discounts" which are often considered a violation of competition law. The buyers may also participate more actively in the determination of the prices, e.g. in principle these could be determined by bargaining between the two sides of the markets.

Finally, the model could be extended to a time horizon longer than two periods. This is not a trivial task, as equilibrium profits may not be possible to be expressed as functions of the capacities in closed form. But in principle this is an interesting extension, as it would allow us to ask how should buyers time their purchases, if the price paths are monotonic, and how the distribution of rents changes as the number of periods increases.

8 Appendix

We now derive another equilibrium in the duopsony model with linear pricing, where the buyers also split their orders in equilibrium. Let the prices set by the two sellers be p_L and p_H and let α_i be the probability that a buyer buys two from seller i , with $(1 - \alpha_L - \alpha_H)$ be the probability that the buyer splits his order

The buyers payoff are as follows. If a buyer buys 2 units from seller L , his payoff is

$$(V_1 + V_2)(1 + \delta) - 2p_L + (1 - \alpha_L)\delta V_3 - (1 - \alpha_L - \alpha_H)\delta E_2, \quad (12)$$

where E_2 is defined as the expected second-period price if one seller sells in period one 3 units and the other 1 unit. If a buyer buys 2 units from H , his payoff is

$$(V_1 + V_2)(1 + \delta) - 2p_H + (1 - \alpha_H)\delta V_3 - (1 - \alpha_L - \alpha_H)\delta E_2. \quad (13)$$

If he splits his order, his payoff is

$$(V_1 + V_2)(1 + \delta) - p_L - p_H + \delta V_3 - (\alpha_L + \alpha_H)\delta E_2. \quad (14)$$

For the buyer to be indifferent among the three alternatives we must have that, combining (12) and (13),

$$\alpha_L = \frac{2(p_H - p_L)}{\delta V_3} + \alpha_H, \quad (15)$$

and, combining (13) and (14),

$$\alpha_H = \frac{V_3(p_H - p_L + \delta E_2) - 4(p_H - p_L)E_2}{(4\delta E_2 - \delta V_3)V_3}.$$

Note that $\frac{\partial \alpha_H}{\partial p_H} = -\frac{1}{\delta V_3}$ and that when $p_L = p_H$,

$$\alpha_H = \alpha_L = \frac{E_2}{4E_2 - V_3}.$$

Now let us turn to the sellers. Expected profit for seller H is:

$$\begin{aligned} \pi_H &= 2\delta V_3 \alpha_L^2 + 4p_H \alpha_H^2 + 2p_H [(1 - \alpha_L - \alpha_H)^2 + 2\alpha_L \alpha_H] + \\ &+ (p_H + \delta V_3) 2\alpha_L (1 - \alpha_L - \alpha_H) + (3p_H + \delta V_3) 2\alpha_H (1 - \alpha_L - \alpha_H) = \\ &= \delta V_3 (2\alpha_L - 4\alpha_L \alpha_H + 2\alpha_H - 2\alpha_H^2) + 2p_H (1 - \alpha_L + \alpha_H). \end{aligned}$$

Dividing by 2, this becomes

$$\delta V_3 (\alpha_L - 2\alpha_L \alpha_H + \alpha_H - \alpha_H^2) + p_H (1 - \alpha_L + \alpha_H).$$

Further, by substituting (15) we have

$$\begin{aligned}\pi_H &= \delta V_3 \left[2\alpha_H + \frac{2(p_H - p_L)}{\delta V_3} - 3\alpha_H^2 - \frac{4\alpha_H(p_H - p_L)}{\delta V_3} \right] + p_H - \frac{2(p_H - p_H p_L)}{\delta V_3} \\ \frac{\partial \pi_H}{\partial p_H} &= \delta V_3 \left[-\frac{2}{\delta V_3} + \frac{2}{\delta V_3} + \frac{6\alpha_H}{\delta V_3} + \frac{4(p_H - p_L)}{\delta^2 V_3^2} - \frac{4\alpha_H}{\delta V_3} \right] + 1 - \frac{2(2p_H - p_L)}{\delta V_3} \\ &= \delta V_3 \left[\frac{2\alpha_H}{\delta V_3} + \frac{4(p_H - p_L)}{\delta^2 V_3^2} \right] + 1 - \frac{2(2p_H - p_L)}{\delta V_3}\end{aligned}$$

Note that if $p_H = p_L = \delta V_3$ and there is no splitting ($\alpha_H = 1/2$) we return to the mixed strategy equilibrium described in Proposition 5.

Now, suppose that the buyers both split their orders with positive probability. Then, at the symmetric equilibrium, $p_H = p_L \Rightarrow \alpha_L = \alpha_H = \alpha = \frac{E_2}{4E_2 - V_3} < 1/2$ and the price charged by both firms in equilibrium is

$$p = \delta V_3 \left[\frac{1}{2} + \frac{E_2}{4E_2 - V_3} \right] < \delta V_3.$$

Each seller's equilibrium profit is then

$$\pi = 2p + \delta V_3(4\alpha - 6\alpha^2)$$

Thus, profits are greater than $2\delta V_3$, thus the firms receive positive rents, but it is less than $\frac{5\delta V_3}{2}$, thus the sellers make fewer profits than when buyers do not split their orders.

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