Pass-through as an Economic Tool*

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Abstract

Pass-through rates play an analogous role for imperfect competition to elasticities when competition is perfect. Log-curvature of demand connects optimal responses to cost and production shocks to the division of gains from trade among consumers, producers and deadweight loss. Therefore in a wide range of oligopoly models simple qualitative properties of the pass-through rate sign, and full knowledge of it, quantifies many comparative statics. Unlike most functional forms for theoretical and empirical

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analysis, our *Adjustable-pass-through* (Apt) demand form and *Constant Pass-through Demand System* (CoPaDS) avoid unjustified ex-ante restrictions on pass-through rates.

Elasticities of supply and demand play a fundamental role in the analysis of competitive markets. Will raising a tax raise or lower revenue? It depends on the elasticities of supply and demand. Envelope conditions of consumer optimization imply (Harberger, 1964; Chetty, 2009) that elasticities drive much of welfare analysis in competitive markets. However Cournot (1838) demonstrated that demand elasticity determines the level, rather than comparative statics, of monopoly and oligopoly prices. Our paper elaborates his argument.

Take the comparative static Cournot most focused on: price with respect to cost. We show this *pass-through rate*[1] is determined by the extent to which imperfect competitors are indifferent over a range of prices, implying that a change in cost significantly shifts their optimal price. Thus the extent of pass-through is determined by the degree to which, at that optimal price, large consumer surplus and deadweight loss tempt the monopolist to change her price. The same factors lead the monopolist to be indifferent over a range of production. In effect, the log-curvature of demand ties responses to cost and production shocks to the division of gains from trade among consumer surplus, producer surplus and deadweight loss.

Pass-through, in turn, is closely tied to how firms respond to changes in demand. Consider a thought experiment proposed by Jeremy Bulow in several recent unpublished discussions. Suppose a dollar subsidy is given to consumers and a dollar tax is imposed on producers. Then the monopolist's inverse demand and marginal costs both rise by a dollar. The neutrality of tax incidence implies that price must rise by a dollar. Therefore the pass-through of cost and demand must sum to one. Thus, with common demand patterns, the sign of cross-firm strategic effects, and through them much of oligopoly theory (Fudenberg and Tirole, 1984; Bulow et al., 1985), is determined by how the pass-through compares to unity.

This paper uses these arguments to simplify, unify, and generalize several areas of the theory of industrial organization. Section I builds the formal foundations of our argument in the context of the simplest monopoly model. Section II illustrates how this logic extends to oligopoly, considering the simple single-product *Generalized Cournot-Stackelberg*

[1] We use "pass-through rate" and "pass-through" interchangeably. This always refers to absolute pass-through not the elasticity of price with respect to cost.

(GCS) models, a generalization of Cournot's classic theories of oligopoly. This GCS model includes quantity-setting competition and price-setting collaboration (perfectly complementary monopoly[2]) with an arbitrary number of firms acting in arbitrary sequence (von Stackelberg, 1934). How pass-through rates compare to unity, and the sign of their slopes in price, at all stages of the production chain predict the comparisons of nearly all mark-ups (or quantities) and profits within and across industrial organizations.

Section III extends Bulow's logic to more general, multi-product oligopoly models with symmetric firms. For such an extension to hold, it must be that changes in competitor or collaborator prices affect inverse demand in the anticipated manner, an assumption we call *horizontal demand* and which Gabaix et al. (2009) show applies to the non-parametric, many-firm version of the Berry et al. (1995) model. They show that log-curvature of assumed distributions of preference parameters determine demand log-curvature. Thus such assumptions sign, through pass-through, most ambiguous comparative statics in oligopoly theory, such as the effects of entry and mergers on other firms' prices. Section IV uses the ideas developed here to simplify, unify, and extend a number of applications beyond classical and neoclassical oligopoly theory that appear elsewhere, including two-sided markets (Weyl, 2009a), merger analysis (Froeb et al., 2005; Farrell and Shapiro, 2008), third-degree price discrimination (Aguirre et al., Forthcoming), price controls in competitive markets (Bulow and Klemperer, 2009) and auction theory (Mares and Swinkels, 2009).

Despite the importance of demand log-curvature in determining, through pass-through, a wide range of outcomes in many models, Subsection V.A shows that functional form and demand shape restrictions, imposed in much empirical and theoretical analysis, restrict it ex-ante in ways that Subsection I.F show are unjustified by existing empirical evidence. We therefore propose, in Subsections IV.B and V.B respectively, multi- and single-product demand forms which maintain extreme tractability while being flexible on pass-through

[2]Our terminology is based on Weyl's best attempt to reconstruct a contemporaneous translation of Cournot's own phrase *concours de producteurs* in other work (Kominers and Weyl, 2009). However what we call the collaboration problem has been called variously "double marginalization", "complementary monopoly" and, recently, "anticommons"; Kominers and Weyl provide a more detailed history.

and, in the second case, its slope. Given the paper's length, Sections II-V are designed to be largely independent of one another, although all build on Section I. Most formal results are established in appendices that appear separately from this paper, along with mathematical software for estimating and manipulating the demand form we propose, at http://www.glenweyl.com.

I. Monopoly Pricing and Pass-through

Consider a monopolist facing[3] smooth, decreasing demand $Q(\cdot)$ and constant marginal cost of production c. The monopolist's familiar first-order condition is :

$$m \equiv p - c = \mu(p) \equiv -\frac{Q(p)}{Q'(p)} = \frac{p}{\epsilon(p)} \tag{1}$$

where $\epsilon(p)$ is the elasticity of demand. We refer[4] to μ as the firm's *market power.* Note that m, to which we refer as the firm's mark-up, is in absolute, not relative, terms: $m = p - c$.

A. Second-order conditions and pass-through

A common condition ensuring the sufficiency of equation (1) for optimization is that demand be log-concave ($\log(Q)'' < 0$), which is equivalent to market power being decreasing. However this condition is grossly sufficient[5] for this purpose. More importantly it restricts the pass-through rate[6], the amount a monopolist finds it optimal to raise prices in response to a small increase in cost. Using implicit differentiation, Cournot showed that a monopolist's optimal

[3]We maintain our (relatively innocuous) assumption of smooth demand and our (quite strong) assumption of constant marginal cost throughout the paper.

[4]Elsewhere it is variously known as the ratio of price to elasticity of demand, the inverse hazard rate and the difference between price and marginal revenue.

[5]For an extensive discussion of the properties of log-concave functions (and particularly probability distributions), see Bagnoli and Bergstrom (2005). The authors also discuss a wide variety of economic applications, including many prominent papers in industrial organization, where log-concavity is assumed.

[6]This is equivalent to marginal revenue sloping down more steeply than inverse demand (Jeremy Bulow).

absolute, not relative ($\frac{dp}{dc}\frac{c}{p}$), *pass-through* of linear cost is given by

$$\rho \equiv \frac{dp}{dc} = \frac{1}{1-\mu'} \tag{2}$$

In what follows, where not explicitly indicated otherwise, we use *pass-through* to refer to the primitive property of demand $\frac{1}{1-\mu'}$, which determines the optimal absolute pass-through rate of a linear cost monopolist, rather than the actual equilibrium pass-through rate in a particular industry. As can easily be seen from equation (2), log-concavity (convexity) is equivalent to pass-through being less (greater) than 1-for-1. We will therefore generally refer to log-concave demand as "cost-absorbing" (e.g. linear) and log-convex demand as "cost-amplifying" (e.g. constant elasticity), using the terminology of Rochet and Tirole (Forthcoming). A much weaker condition than cost absorption that makes equation (1) sufficient for the monopolist's optimization is that $\mu'(p) < 1$ for all p. This condition is equivalent to marginal revenue declining in quantity, so we will refer to it as "declining marginal revenue" (DMR). It is also known in the literature, variously, as -1-concavity of demand (it is equivalent to $\frac{1}{Q}$ being convex) and "regularity" (Myerson, 1981). The main testable implication of this assumption is that a firm facing a binding price control will choose to charge at the controlled price.

Theorem 1. *If demand exhibits DMR then any solution to equation (1) is the monopolist's optimal price and for any cost a monopolist facing price ceiling (floor) below (above) her unconstrained optimum will always choose to charge a price at that ceiling (floor). Conversely if Q fails to satisfy weak DMR, even at single point, then for some cost*

1. *there is a solution to (1) which is not optimal.*

2. *there is a price ceiling (or floor) below (above) the monopolist's unconstrained optimal price given that cost such that the monopolist chooses a constrained price strictly below (above) that ceiling (or floor).*

Proof. See appendix *Monopoly* Section I.

DMR is the weakest condition[7] ensuring global "first-orderness" for any cost.

B. Pass-through as an elasticity

If a monopolist faces a rather rigid "price the market will bear" then her optimal price is very sharply defined and increases in cost will hardly affect her optimal price. On the other hand, if the monopolist is close to indifferent between a range of prices, a small increase in cost can cause a dramatic shift in optimal price[8]. This can be seen formally by noting that (at the monopoly optimal price)

$$\rho = \frac{1}{-\frac{d^2\pi}{dm^2}\frac{m^2}{\pi}} \tag{3}$$

Thus pass-through is exactly the inverse of the second-order elasticity of profits[9] with respect to mark-up. This is a crisp expression of Cournot's argument: in choosing their optimal price, monopolists take first-order effects (elasticities of demand) into account. Therefore the *level* of price elasticity is replaced by its own *elasticity*.

The fact that pass-through, like other elasticities, is a unit-less measure, rather than a derivative, implies that it also determines the comparative statics of the monopolist's

[7]Because it has no grounding in consumer theory it is a strong restriction on demand functions. It is, however a reasonable methodological commitment for two reasons.

First, it amounts to a natural extension of simplifying assumptions made for tractability, such as differentiability of demand. Many results derived on the basis of such assumptions can easily be generalized (Milgrom and Shannon, 1994; Amir and Grilo, 1999; Amir and Lambson, 2000) and they are therefore typically technical conveniences rather than substantive restrictions.

Second, in empirical applications data over a limited range is commonly used for estimation. It seems difficult to make predictions about discontinuously different outcomes based on such data. An analyst would require a estimate of third-order properties of demand, so that the rate at which firm best response curves return to a fixed point could be measured. Thus the plausibility of structural empirics may implicitly rest on these assumptions of endogenous continuity of firm behavior, based on unimodality of consumer valuations. Whether or not such an approach is plausible in most markets is largely an open question, although one that is empirically testable by Theorem 1. Evidence of multi-modality in the distribution of consumer preferences in supermarkets presented by Burda et al. (2008), based on more flexible, non-parametric estimation than is typically employed, is therefore worrying.

[8]Joe Farrell suggests that when pass-through is high it may also be unpredictable if firms are imperfect profit maximizers. One way to think about pass-through is as the inverse of the friction on a plane. Costs gives the firm a shove and it slides longer, but also less predictably, the higher pass-through is.

[9]We suspect that, given the role of second derivatives in statistical discrimination problems (Fisher, 1922; Chernoff, 1959), that the pass-through is likely related to the optimal degree of price experimentation by a monopolist. An interesting topic for future research is to understand the relationship between pass-through and experimentation, as this may provide a way of generating testable implications of optimal experimentation, either with or without rational expectations.

production. Imagine a monopolist choosing an optimal quantity to produce, given that there exists some exogenous quantity[10] $\tilde{q}$ of the good already available. Let $q^\star \equiv \tilde{q} + q_M$, the monopolist's optimal production, be the total industry production given that the monopolist optimizes. The natural quantity analog of the pass-through rate is the *quantity pass-through*[11]

$$\rho_q \equiv \frac{dq^\star}{d\tilde{q}} \tag{4}$$

Theorem 2. $\rho(p^\star) = \rho_q(q^\star)$ *when the optima are for the same values of* c *and* $\tilde{q}$.

Proof. A firm facing exogenous quantity $\tilde{q}$ is earns profits $(q^\star - \tilde{q})(P[q^\star] - c)$ where $P = Q^{-1}$. The marginal revenue of raising $q^\star$ is $P - c + P'(q^\star - \tilde{q})$. Thus effectively an increase of $d\tilde{q}$ in $\tilde{q}$ is equivalent to a decline in her marginal cost by an amount $-P'd\tilde{q}$. Thus it must cause prices to rise by $\rho P' d\tilde{q}$ and thus quantity rise by $Q'P'\rho d\tilde{q} = \rho d\tilde{q}$. Thus $\rho_Q = \rho$.

Figure 1 gives some graphical intuition. The first three panels in the figure show two different constant pass-through demand functions, one (linear) with a low pass-through rate and one (translated constant elasticity) with a high pass-through rate that share a common monopoly optimal price and demand at that price $(q^\star, p^\star)$. The first two panels demonstrate that a change in price or quantity away from this optimum will lead profits to die off much more quickly when pass-through is low than when it is high. The third panel shows the associated marginal revenue curves for each demand and the price increase for each resulting from a small increase in cost (from 0), showing how high pass-through results from slow fall-off in profits as prices change.

[10] While this "exogenous quantity" may appear an artificial construct, it has, as shown below, a very natural interpretation in the context of quantity competition. In particular$\rho_q - 1$ determines the strategic complementarity (if it is positive) or substitutability (if negative) in quantity competition. The first work to (implicitly) link pass-through rates and the strategic interactions in Cournot competition was Seade (1986).

[11] In appendix *Apt Demand* Subsection II.B we briefly discuss the equivalent of DMR for the production problem, which we assume in analyzing the production problem.

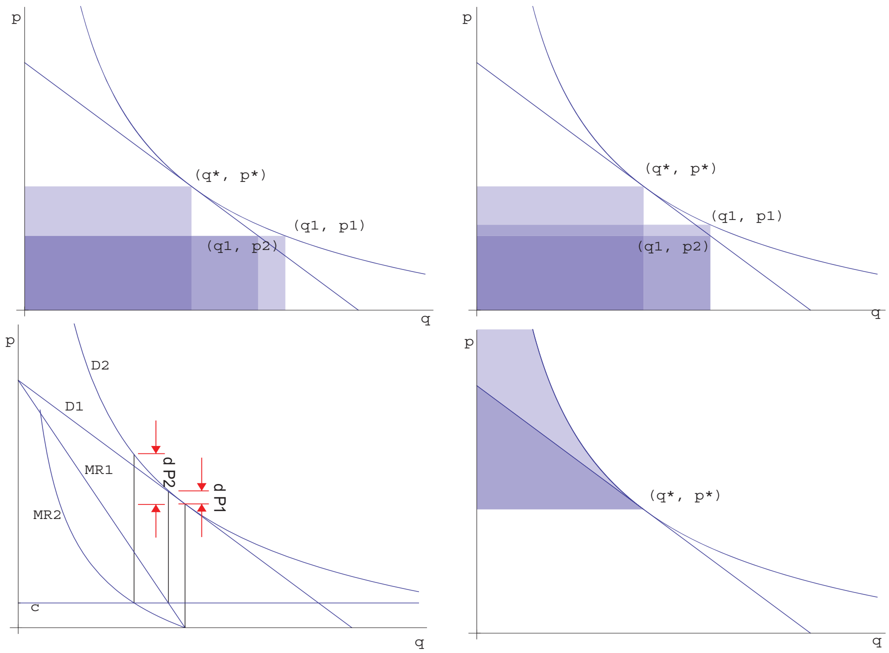

Figure 1: Low $Q_1(p) = 1 - p$ and high $Q_2(p) = \frac{2}{(1+2p)^2}$ constant pass-through demands. High pass-through leads profits to die off more slowly than low if either price (Panel 1) or quantity (Panel 2) changes away from the optimum. It also leads pass-through of an increase in cost to price to be higher, as seen by drawing marginal revenue curves (Panel 3). Finally it leads to higher consumer surplus as shown in Panel 4.

C. Pass-through and the division of gains from trade

Monopolists are nearly indifferent over a range of prices if there is consumer surplus and deadweight losses are large. She will then be conflicted between chasing this surplus or deadweight loss by charging a higher or lower price and continuing to serve her optimal market. The pass-through rate is therefore closely related to the division of potential gains from trade among consumer surplus, producer surplus and deadweight loss[12]. Figure 1 provides some graphical intuition in the special case of constant pass-through demand.

Formally, when price p is charged and $\overline{p}$ is supremum of prices (possibly ∞) at which

[12]This reasoning also suggests, as pointed out to us by colleagues at the University of Chicago, the only reason we know of why we may not expect to find high pass-through rates in industries with limited price discrimination. If consumer surplus is large relative to profits, firms are liikely to try to price discriminate to capture some of that surplus. We may therefore expect that industries, which would have high pass-through rates under uniform pricing, will not use uniform pricing at all. This suggests further investigation of the relationship between pass-through rates and price discrimination and its empirical implications, a theme we occasionally return to in footnotes below.

demand is strictly positive, consumer surplus $V(p) \equiv \int_p^{\overline{p}} Q(q)dq$ and producer surplus, the monopolist's profits, is $\mu(p)Q(p)$ by her first-order conditions. Therefore the ratio of consumer to producer surplus is $r(p) \equiv \frac{V(p)}{Q(p)\mu(p)}$ at the profit maximizing price. Dually, when production is $q < \overline{q} \equiv Q(c)$ deadweight loss is $d(q) \equiv \int_q^{\overline{q}} m(q)dq$ where $m(q) \equiv P(q) - c$ and profits can be written as $\kappa(q)m(q)$ where the *market capacity* $\kappa(q) \equiv -\frac{m(q)}{m'(q)}$ as, by duality with the price optimization, $q^\star - \tilde{q} = \kappa(q^\star)$. Thus the ratio of deadweight loss to producer surplus (at profit-maximizing production) is $r_Q \equiv \frac{d(q)}{\kappa(q)m(q)}$

The following theorem states that this division of surplus is given by an average of the pass-through rate over prices above the monopolist's optimal price.

Theorem 3. *Assume DMR and* limit DMR*, that* $\lim_{p\to\infty} \mu'(p) < 1$*, as well as the equivalent conditions on* $\kappa(q)$*. Then*

$$r(p) = \overline{\rho}(p) \equiv \int_p^{\overline{p}} \lambda(x;p)\rho(x)dx \quad (5)$$

$$d(q) = \overline{\rho_Q}(q) \equiv \int_q^{\overline{q}} \lambda_Q(x;q)\rho_Q(x)dx \quad (6)$$

where $\lambda(x;p) \equiv \frac{\nu(x)}{\int_p^{\overline{p}} \nu(y)dy}$, $\nu(p) \equiv \frac{Q(p)}{\rho(p)}$, $\lambda_Q(x;q) \equiv \frac{\omega(x)}{\int_q^{\overline{q}} \omega(y)dy}$, $\omega(q) \equiv \frac{m(q)}{\rho_Q(q)}$ *thus*

$$\int_p^{\overline{p}} \lambda(x;p)dx = \int_q^{\overline{q}} \lambda_Q(x;q) = 1$$

Proof. See appendix *Monopoly* Section III.

This result has several immediate corollaries[13]. If (quantity) pass-through stays above a

[13]These corollaries, which together we believe (but have not shown) imply Theorem 3, were independently discovered by Mares and Swinkels (2009), who use them as the foundation of a comprehensive analysis of first-price, asymmetric auctions that we describe in Subsection IV.E. Relative to this paper, Mares and Swinkels also provide a much fuller treatment of the mathematical context and implications of these results. For example, they discuss how the result makes quantitative the classic qualitative link (Prékopa, 1971; An, 1998) between log-curvature of a distribution and the log-curvature of its survivor function (the fatness of its upper tail). Theorem 7 in appendix *Monopoly* Section III emphasizes this: if $\overline{p}$ is finite (demand is "tailless") demand is cost-absorbing above some price and if $\overline{p} = \infty$ (consumer values are unbounded above) pass-through cannot be bounded below 1 for large prices. Malueg (1994) established the special case of the first corollary when $k = \frac{1}{2}$ (ie the distinction between concave and convex demand) in an elegant graphical analysis.

threshold k, then so does the consumer-to-producer surplus (consumer surplus-to-deadweight loss) ratio[14]. If (quantity) pass-through is increasing then the consumer-to-producer surplus (consumer surplus-to-deadweight loss) ratio is always above local (quantity) pass-through.

The result might also be used as a means of guessing, based on local data, the magnitude of consumer surplus or deadweight loss. Logic runs in the other direction as well: monopoly[15] pass-through is high to the extent that the distribution of consumer valuations is fat-tailed.

Such connections are valid only if local pass-through rates approximate their global averages. Conversely, if one believes pass-through is typically quite variable, one should reject demand functions typically used, as they nearly all have pass-through rates that are on the same side of unity globally and many common demand functions (linear, constant elasticity, negative exponential) actually have constant pass-through (see Section V).

D. Pass-through of demand

As alluded to in the introduction, Jeremy Bulow has recently argued firms' response to change in the demand they face should be closely connected to their cost-price pass-through. While we independently arrived at a similar conclusion in a prior draft of this paper, we prefer and therefore present his formulation of the basic argument.

Consider the monopolist's problem formulated as a choice of quantity. She equates marginal cost to marginal revenue: $P(q^\star) + P'(q^\star)q^\star = c$. Suppose that both her inverse demand P and marginal cost c increase by one dollar; clearly $P(q^\star) + 1 + P'(q^\star)q^\star = c + 1$ and thus her optimal production is unchanged. Therefore an increase by one dollar in both her inverse demand and marginal cost must raise her price by exactly one dollar. Thus under

[14]E.g. globally cost-absorbing demand has greater producer than consumer surplus at profit maximization. See our conclusion for a more general statement of this conjecture.

[15]Beyond monopoly it applies in a common oligopoly context considered in the empirical literature on the surplus generated by new products. If we consider (Hausman, 1997; Gentzkow, 2007) the surplus created by a new good, holding fixed the prices of all other goods, then the formula is valid under oligopoly so long as the pass-through rate is the primitive rate driven by the log-curvature of demand, the *short-term own pass-through* we discuss in Subsection III.A. Furthermore we suspect, but have not formally shown, that the basic intuitions of the result extend beyond linear cost monopoly. For example, increasing marginal costs will tend to both reduce pass-through, as increases in price trigger reduction in production and therefore cost, and raise profits relative to deadweight loss and consumer surplus.

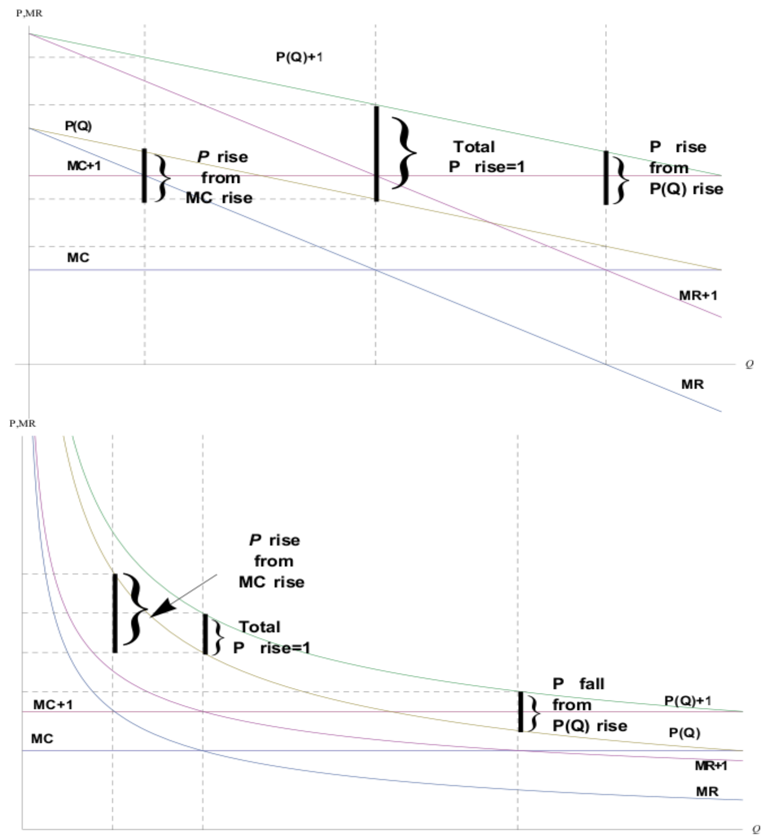

Figure 2: Pass-through of demand and cost shifts with the linear and shifted-constant elasticity demand curves of Figure 1. A dollar increase in both inverse demand and marginal cost $\implies$ dollar increase in price. Under linear this is split evenly between the two; with CE the cost-driven price rise is large, but the demand rise lowers price.

cost absorption, an increase in inverse demand increases price, as common intuition runs. Under cost amplification, in contrast, an increase in inverse demand leads to a decrease in optimal price. This argument is central to our theory of multi-product industries in Section III.

Figure 2 serves as a graphical representation. For two different demands, linear and constant elasticity, inverse demand and in turn marginal revenue and marginal cost are increased by a dollar. While with linear demand both the cost and demand raise prices, each by fifty cents, under constant elasticity the rise in demand decreases price while the rise in cost increases it by more than a dollar. For both the net effect is a dollar increase in price.

E. Signed Pass-through Assumptions

For all these reasons it seems natural to assume that demand is globally either cost-absorbing or cost-amplifying. In fact, an even more robust feature of demand functions commonly used is that their pass-through rates are strictly monotone (or constant) in price, suggesting also that such monotonicity is a natural, weak restriction to place on demand functions. We maintain these *Signed Pass-through Assumptions* (SPAs) throughout the paper[16].

Assumption 1. *Demand is either globally cost-absorbing (log-concave), globally cost-amplifying (log-covex) or globally constant mark-up (log-linear). Demand has either globally increasing pass-through (concave inverse hazard rate), globally decreasing pass-through (convex inverse hazards) or constant pass-through (linear inverse hazards) as a function of cost/price.*

F. Empirical evidence on pass-through

Whether relaxing log-concavity of demand is a mere theoretical exercise or empirically crucial depends on whether pass-through rates greater than unity are common in real oligopolies. Empirical studies based on imposed functional forms are of little use as the two most common functional forms used in the monopoly case, linear and constant elasticity, assume ex-ante that demand is respectively cost-absorbing and cost-amplifying. For example, Baker and Bresnahan (1988) famously estimated the residual demand curve of a monopolist, exactly what we are interested in, but assume a constant elasticity form as they were interested in demand elasticity, rather than pass-through. Unfortunately reduced-form evidence on pass-

[16]Section V shows these are satisfied by nearly all common statistical distributions and demand functions. Whether this calls into question typical demand functions and distributions, or bolsters Assumption 1, is left for the reader to decide. The full force of the assumption is not needed for most results: it can typically be replaced by assuming properties about some average of pass-through rates (or their slope) over some relevant range of prices. These weaker assumptions can be seen as relatively mild strengthening of boundedness of higher-order effects conditions used to justify traditional linear approximation methods used throughout economics. However, since the exact assumptions needed for particular results vary in a confusing way across examples if maximally relaxed, for expositional clarity we maintain Assumption 1 throughout.

through rates is sparse and where it exists it is rarely firm-specific, plausibly in an industry where strategic interactions can (justifiably) be ignored or for firms with a single product.

Barzel (1976) reports cigarette industry pass-through rates rather than firm (much less monopoly) pass-through rates. Section III argues that these may plausibly be on the same side of 1. Therefore, Barzel's finding that pass-through rates are significantly above 1 provides some evidence for cost amplification. The pass-through of broader sales taxes have been extensively studied. Haig and Shoup (1934) interviewed shop-owners and found they reported cost absorption, while more quantitative studies by Besley and Rosen (1998a,b) found cost amplification and Poterba (1996) found essentially constant mark-ups. In a long-term study of the sugar industry in the late nineteenth century, Genesove and Mullin (1998) found very slight cost amplification albeit in a very competitive industry (see Subsection III.A below). Exchange rate shocks are typically partially absorbed in the short run, but close to fully passed-through in the longer-term (Menon, 1995; Campa and Goldberg, 2005).

Evidence on product-and-firm specific pass-through rates is similarly mixed. Ashenfelter et al. (1998) and Besanko et al. (2001) find significant cost absorption of firm-specific cost shocks, but pass-through is non-negligable, ranging form 15-40% as an elasticity (always below pass-through rates if there is positive market power). In the context of multi-product semi-monopoly (a major retailers), the informal, accounting-based study of Chevalier and Curhan (1976) found a mix of cost absorption, constant mark-ups and cost amplification while the econometric approach of Besanko et al. (2005) reports that on approximately 30% of products own-brand pass-through elasticities are above unity.

There appears to be little empirical evidence and at best weak theoretical arguments pointing towards pass-through rates being typically below one. Our guess is that cost absorption is somewhat more common than cost amplification, but cost amplification hardly seems rare. As far as we know there is no evidence on how pass-through rates change with prices. Thus, at least at this stage, theory should accommodate all possibilities allowed by Assumption 1 and results derived from assumptions restricting these cases, as made by common demand forms (Subsection V.A), are implausible absent further empirical evidence. More such evidence is clearly needed.

II. The Generalized Cournot-Stackelberg Model

This section studies applications of pass-through to generalizations of the single product models studied by Cournot to all sequential actions in the spirit of von Stackelberg (1934).

A. Cournot collaboration

The classic Cournot collaboration model has two equivalent formulations. In the first (Cournot, 1838), two monopolists sell goods that are perfect complements in consumption. In the second (Spengler, 1950), one firm sells an input to a second firm which sells to a consumer. The only difference between these is that in Cournot's the assembly is performed by the consumer and in Spengler's it is performed by the downstream firm. The firms have total linear cost between them c_I, the division of which we will show is irrelevant to the outcomes of interest. The natural benchmark against which to judge the vertically separated organizations is that of a single vertically integrated monopolist Integrated who sets her markup $m_I^\star$ to solve equation (1), plugging in c_I for c.

The separated organization envisioned by Cournot is shown in Figure 2. The two firms simultaneously choose prices to charge consumers for whom the goods are perfect complements in consumption. The first-order condition for each firm i is

$$m_i^\star = \mu(m_i^\star + m_j^\star + c_I) \tag{7}$$

At equilibrium each firm earns profits $\pi^\star$ and the total mark-up in the industry is $m_N^\star \equiv 2m^\star$.

Cournot's problem can also be formulated in Spengler's physical organization, shown in Figure 3, so long as the firms chose their mark-ups simultaneously. Thus it is the Nash timing that distinguishes the situation in Figure 3 (which we therefore call the Cournot-Nash

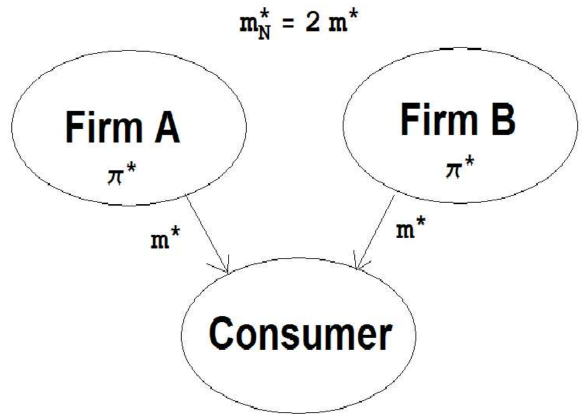

Figure 3: The Nash industrial organization

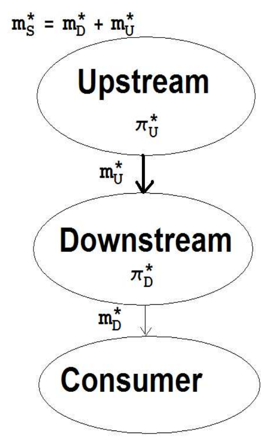

Figure 4: The Spengler-Stackelberg industrial organization

organization) from that pictured in Figure 4 where, as Spengler originally assumed, Upstream commits to its price before Downstream chooses its price. Again, Cournot's physical organization combined with price leadership in the spirit of von Stackelberg (1934) by one will yield the same outcomes as the organization in Figure 4. For any choice of mark-up m_U by Upstream, the optimal mark-up of Downstream is given, as Upstream's mark-up is analogous to an increased cost for Downstream, by

$$m_D^{\star} = \mu(m_U + m_D^{\star} + c_I) \tag{8}$$

Anticipating this, Upstream maximizes profits $m_U D\big(m_U + m_D(m_U) + c_I\big)$ according to the first-order condition

$$m_U^\star = \frac{\mu\left(m_U^\star + m_D(m_U^\star) + c_I\right)}{\rho\left(m_U^\star + m_D(m_U^\star) + c_I\right)} \tag{9}$$

Equation (9) resembles Downstream's first-order condition, but takes into account the strategic effect of Upstream's choice on Downstream. Under cost absorption ($\mu' < 1$), mark-ups are strategic substitutes in the sense of Bulow et al. (1985): the increase in mark-up by one firm, an action equivalent to imposing a tax on the other firm, induces the other firm to absorb this increase and lower its mark-up. Conversely, under cost amplification, mark-ups are strategic complements. At equilibrium Upstream earns profits $\pi_U^\star$, the Downstream earns profits $\pi_D^\star$ and the total markup charged by the two firms is $m_S^\star \equiv m_U^\star + m_D^\star$.

The classic theory[17] of the collaboration problem states that a merger to monopoly from either separated organization reduces total industry prices and increases total industry profits as firms internalize the negative externalities across firms of high prices. This says nothing, however, about comparing either the two separated organizations or the mark-ups or profits of individual firms within or across organizations. In fact, $4! = 24$ rankings of firm mark-ups, $3! = 6$ rankings of firm profits, 2 rankings of Stackelberg versus Nash mark-ups, and 4 ranges of values for pass-through rates and slope are possible. However as Table 1 summarizes, simply imposing SPAs narrows these 1152 possible patterns to only 4: if we know whether demand is cost-absorbing or cost-amplifying and whether pass-through is increasing or de-

[17] The theory was originally developed by Cournot and Spengler, but for a general modern treatment see Bergstrom (1978). Kominers and Weyl (2009) connect it to the corresponding mechanism design problem. The relationship between these two suggests that the relatively special results developed, like those of Cournot, may have broader relevance to market design. There are a number of other examples of this broader relevance of our results. On the empirical side, if industrial organizations shift and pass-through can be measured, as in Mortimer (2008), they can be used to test the model. Such testing has been a topic of substantial interest in recent years (Villas-Boas, 2007) and has been forced to rely on restrictive (convex, but cost-absorbing) functional form restrictions for identification, a restriction that our results potentially relax. Pass-through can be used to determine policymakers' preferred organization as well as to predict the organization likely to emerge endogenously(Amir and Grilo, 1999). The reasoning can be adapted (Subsection IV.B) to analyze mergers between firms that produce complements in some markets and substitutes in others. Furthermore, they can be used to analyze vertical relationships among tax authorities (see Keen (1998) for a survey) or the tax relationship between a regulator and a monopolist, an application suggested to me by Bill Rogerson. Additionally, they can be used to analyze the effects of commitment on the pricing of monopolists selling goods with inter-temporal complementarities (Klemperer, 1987; Murphy and Becker, 1988), an application suggested to me by Kevin Murphy. Finally, Martimort and Stole (2009) establish a tight connection between delegated common agency games and the vertical monopolies problem.

	$\rho < 1$		$\rho > 1$	
	Cost absorption		Cost amplification	
	Decreasing pass-through		Decreasing pass-through	
	$m_U^\star$		$m^\star$	
ρ'	$\vee$	$\pi_U^\star$	$\vee$	$\pi_D^\star$
$\wedge$	$m_I^\star < m_N^\star < m_S^\star$	$\vee$	$m_D^\star$	$\vee$
0	$\vee$	$\pi^\star$	$\vee$	$\pi_U^\star$
	$m^\star$	$\vee$	$m_U^\star$	$\vee$
	$\vee$	$\pi_D^\star$	$\vee$	$\pi^\star$
	$m_D^\star$		$m_I^\star < m_S^\star < m_N^\star$	
	Cost absorption		Cost amplification	
	Increasing pass-through		Increasing pass-through	
	$m_I^\star < m_N^\star < m_S^\star$		$m^\star$	
ρ'	$\vee$	$\pi_U^\star$	$\vee$	$\pi_D^\star$
$\vee$	$m_U^\star$	$\vee$	$m_D^\star$	$\vee$
0	$\vee$	$\pi^\star$	$\vee$	$\pi_U^\star$
	$m^\star$	$\vee$	$m_I^\star < m_S^\star < m_N^\star$	$\vee$
	$\vee$	$\pi_D^\star$	$\vee$	$\pi^\star$
	$m_D^\star$		$m_U^\star$	

Table 1: Comparing mark-ups and profits among firms within and across organization of the Cournot-Spengler double marginalization model

creasing in cost, we obtain a full ranking of firm and industry mark-ups and profits within and across industrial organizations. Furthermore, it is easy to show[18] that if the first-order effects of cost shocks on prices can be estimated in the Spengler-Stackelberg organization, or second-order effects[19] in the Nash organization, a unique prediction is identified.

Most of the action in Table 1 happens across the vertical line, moving from cost absorption and thus strategic substitutes to cost amplification and thus strategic complements. We now intuitively describe the logic driving this result.

Does a Nash firm or Integrated charge a higher mark-up? The only difference between their incentives is that a Nash firm's consumers taxed by the collaborator's markup. If it is optimal for her to absorb the tax, the Nash firm will choose a lower mark-up than Integrated; if it is optimal to amplify it, Nash will charge a higher mark-up than Integrated.

Does Upstream or Downstream charge a higher mark-up? Both face the same demand

[18]See appendix *Generalized Cournot-Stackelberg Models* Section II.

[19]First-order effects suffice to identify all comparisons but $m^{I^\star}$ versus $m^{U^\star}$.

and therefore have the same market power. The only difference in their incentives is that Upstream's mark-up affects Downstream choice, giving Upstream an incentive to do whatever induces Downstream to reduce her mark-up. When mark-ups are strategic substitutes (complements) this involves Upstream increasing (decreasing) her mark-up[20].

All of the rest of the results that shift across the vertical line follow similar logic. The one comparison that varies across the horizontal line is between $m_U^\star$ and $m_I^\star$. This difference is closely related to the fact that pass-through, rather than elasticity, determines the comparative statics of monopoly: in the Stackelberg organization the effects of cost changes are filtered through *two layers* of firm optimization, so third-order properties of demand become relevant. However this also means that under the Spengler-Stackelberg equilibrium, the slope of pass-through is observable in the first-order pass-through behavior of the upstream firm[21].

More precisely, in the case of cost absorption, there are two incentives facing the Upstream firm. On the one hand she would like to increase her mark-up, relative to what Integrated would charge, as she has a strategic incentive to induce Downstream to decrease her mark-up. On the other hand Upstream has an incentive to partially absorb Downstream's mark-up which Integrated does not confront; this leads Upstream to decrease her mark-up relative to what Integrated would charge. The first strategic incentive is marginal: by how much, on the margin, does a small increase in Upstream's price induce Downstream to reduce her price? The second incentive, on the other hand, depends on the average rate at which Upstream should absorb Downstream's mark-up. Unsurprisingly, the relative size of the average versus the marginal effect depends on whether pass-through is increasing or decreasing in cost.

[20]Because both face the same end-demand, this further determines the comparison of their profits.

[21]This point was first made to us by Kevin Murphy and, as shown in appendix *Generalized Cournot-Stackelberg Models* Section II, extends to the GCS model: the information about pass-through rates that one needs to sign comparisons within and across industrial organizations are empirically identified as long as one can observe the the first-order effects of cost on prices at all layers starting from the organization with the largest number of layers.

B. The GCS collaboration model

For expositional clarity we focused in the previous subsection on the two firm case. However our results generalize to an arbitrary number of firms acting in arbitrary sequence.

There are a number of collaborators, each monopolizing the production of a component of an aggregate good, divided into K groups. Each group $k = 1, \ldots, K$ has N_k firms. The total linear cost of production of all firms is c. In the first of K rounds of the pricing game, the N_K firms in group K simultaneously choose their mark-ups m_{i_K} for $i_K = 1, \ldots N_K$. In the kth round, for $k = 2, \ldots, K-1$ the N_{K-k+1} firms in group $K-k+1$ choose their mark-ups taking as given the mark-ups of all firms in groups $k' > K-k+1$. Finally in round K all firms in group 1 simultaneously choose their mark-up and each firm i_k receives a payoff $\pi_{i_k} = m_{i_k} Q\left(c + \sum_{k=1}^{K} \sum_{i=1}^{N_k} m_{i_k}\right)$. The basic set-up is pictured in Figure 5[22]. We refer to this model as the *Generalized Cournot-Stackelberg (GCS) collaboration model*. As far as we know this general model is novel to this paper and we are there first to provide any general results on it. It nests all of the common industrial organizations of vertical monopolies. For example, the Spengler-Stackelberg organization from the previous section is the special case of $K = 2, N_k = 1$ and the Cournot-Nash organization when $K = 1$ and $N_1 = 2$.

Because the results for the GCS vertical monopolies model are so general, our broad theorems on it are somewhat cumbersome formal statements. Therefore instead of describing them in detail here, we have left their formal statement and proof to Appendix *Generalized Cournot-Stackelberg Models* Section I. Instead we discuss the results informally, showing how they naturally generalize those for the simple two-firm case[23].

[22]For example, in the case when $N_k = 1$ for all k, one considered by Anderson and Engers (1992) in the case of quantity competition, this game is often thought of as firm 1_K selling an input to firm 1_{k-1} who transforms this input into another intermediate input and sells this to firm 1_{k-2} and so forth until it reaches firm 1_1 which produces a final output selling it to a final consumer.

[23]Like the two-firm case, most of the comparisons here involve comparisons across two levels of the chain or comparisons across industrial organizations differing only in that firms have changed positions across two levels of the chain. For results that go much beyond these, one would need to combine the pass-through rates at various levels of the chain or perhaps even consider higher-order properties of pass-through. We conjecture, but given space constraints have not attempted to show, that it is possible to rank all mark-ups and profits within and across industrial organizations by observing the effects of a shock to cost on prices at all levels in whichever industrial organization being compared has more levels.

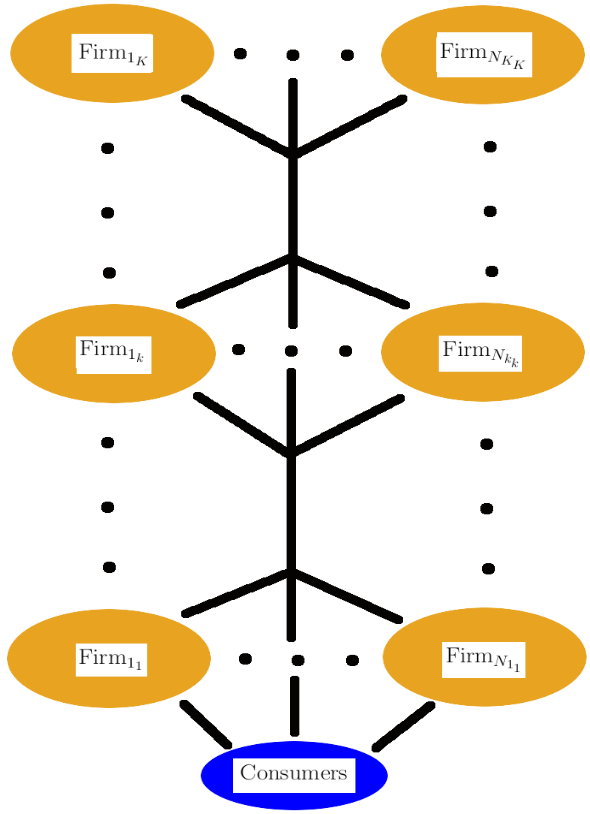

Figure 5: The GCS vertical monopolies model

Let $p_k^\star$ be the total price after the mark-ups of all firms weakly above k have been included; thus $p_1^\star$ is the final price to consumers and the total cost of all firms is $p_{K+1}^\star = c$. Let $\rho_k \equiv \frac{dp_k^\star}{dp_{k+1}^\star}$ and assume that the SPAs[24] apply to ρ_k for all k. Let $m_k^\star$ be the equilibrium mark-up of a firm at the kth level. Results follow the two-firm comparisons they generalize.

- $m_I^\star$ v. $m^\star$: Entry of a firm at level K will lead to higher (lower) $m_K^\star$ if $\rho_K > (<)1$.
- $m_U^\star$ v. $m_D^\star$: $m_{k+1}^\star > (<) m_k^\star \iff \rho_k < (>)1$.
- $m_I^\star$ v. $m_U^\star$: Suppose there is a lone "leader" firm acting in group K. If $\tilde{N}$ firms now act after the leader but before all other firms, the leader charges mark-up $\tilde{m}_{K+1}^\star > (<) m_K^\star$

[24]That is we assume that each ρ_k stays globally on one side of unity and each is globally monotone, though they need not be consistent across different k.

in the new equilibrium if and only if $\rho'_K < (>) 0$.

- $m^\star_U$ v. $m^\star$: A firm moving from level K to level $K+1$, with $N_K > 1$, charges a higher (lower) mark-up after the move if and only if $\rho_K < (>) 1$.

- $m^\star_S$ v. $m^\star_N$: Imagine moving one of the N_k firms currently acting at level k up to acting with the firms at level $k+1$ for any k with $N_k > 1$. This will lead to higher (lower) final prices to consumers[25] if and only if $(\rho_k - 1)(N_k - 1 - \rho_k N_{k+1}) < (>) 0$.

Each ρ_k, as well as ρ'_k for all $k < K$, can be observed if there is a shock to cost at any level by observing what happens to prices moving between levels. In fact, the comparison of mark-ups of firms at various stages (bullet 2 above) immediately identifies whether demand is cost-absorbing or cost-amplifying at all stages below K.

C. GCS quantity competition

Sonnenschein (1968) argued that Cournot collaboration is just the dual of (symmetric linear cost) Cournot competition: in the former the quantity is a function of the sum of mark-ups, while in the second the mark-up is a function of the sum of quantities. Therefore all our results[26] above apply equally to the quantity competition GCS model, where groups of firms sequentially choose production levels, *mutatis mutandis*: mark-ups become quantities, quantities mark-ups, pass-through becomes quantity pass-through, etc. Quantity pass-through rates can also be measured qualitatively by comparing relative productions of firms and/or quantitively using exogenous industry cost shocks appropriately (see Appendix Section III).

[25] Note that when $k = K$, $N_{k+1} = 0$ so, as in the two-firm case, cost-abosrption vs. amplification on its own determines whether sequentiality or simultaneity leads to higher prices.

[26] Some of these results, in the special case of two firms and the context of quantity competition, were first established by Dowrick (1986), Amir and Grilo (1999) and Amir and Lambson (2000), though the connection to pass-through was not recognized.

III. Multiple Products

This section invokes Bulow's argument, discussed in the introduction, to extend our results to industries with multiple products, as these are of interest in many applications. Here we discuss only symmetric linear cost oligopolies (with either substitutes or complements). While this setting is restrictive, it is dominant in the recent theoretical literature on pass-through with multiple firms (Anderson et al., 2001). Our focus is to show how the basic connection between cost and demand pass-through can be applied to oligopoly theory (Subsection A), as well as to show a example of how this insight might be applied to a simplified version model of potential empirical interest (Subsection B).

A. Horizontal demand systems

For Bulow's insight to bear directly on oligopoly interactions, changes in oligopolists' prices must shift other firms' inverse demand curves in the directions one would expect: an increase in the price of a substitutable (complementary) good must raise (lower) inverse demand. The simplest case in which this holds is when such changes trigger a uniform upward (downward) shift in inverse demand. We refer to demand systems with this property as *horizontal*. If competition is in quantities a la Cournot the natural analog of this assumption is that an increase in the production of a substitutable (complementary) good lowers (raises) demand uniformly. While all results below have natural analogs for that setting, we primarily focus on the more common assumption in multi-product models of Nash-in-prices competition.

Formally, consider an industry with N firms each producing a single good and let $\mathbb{Q} : \mathbb{R}^N \to \mathbb{R}^N$ be the (smooth) demand function, with the interpretation that if firm i charges price p_i and other firms charge prices $\mathbf{p}_{-i}$ demand for i's goods will be $Q_i(p_i, \mathbf{p}_{-i})$.

Definition 1. $\mathbb{Q}$ *is a* symmetric horizontal demand system *(SHDS) for a Bertrand oligopoly if for all* i $Q_i(p_i, \mathbf{p}) = \tilde{Q}\left(p_i - g[\mathbf{p}_{-i}]\right)$ *for some* $g : \mathbb{R}^{N-1} \to \mathbb{R}$ *and* $\tilde{Q} : \mathbb{R} \to \mathbb{R}$.

Symmetric horizontal inverse demand is defined *mutatis mutandis* for Cournot (quantity) oligopoly by the Cournot-Bertrand duality (Singh and Vives, 1984). HDSs include the linear demand system and a generalization of this, Horizontal Constant Pass-through Demand System we propose in Section IV.B. However HDSs are much more general than HCoPaDS as it imposes no functional form on $\tilde{Q}$ nor on g.

In additional to horizontality and symmetry, we assume that Q has small enough income effect that it would have a unique general equilibrium under perfect competition, as well as conditions on demand so that oligopoly equilibrium is unique and "first-order"[27].

Letting $\mathbf{p}^\star$ be the (unique, symmetric) equilibrium price vector given symmetric constant marginal costs c, there are four natural notions of pass-through in this context:

1. *Short-run own* (Sop): this is the effect of an increase in one firm's cost, holding other firm prices fixed: $\rho_i \equiv \left.\frac{\partial p_i^\star}{\partial c_i}\right|_{\mathbf{p}_{-i}\text{constrained}}$. It is always positive and is the same as the pass-through rate if the industry were cartelized or merged to monopoly.

2. *Long-run own* (Lop): this is the effect of an increase in one firm's cost on its own equilibrium price $\rho_i^{eq} \equiv \frac{dp_i^\star}{dc_i}$. Stability ensures this is positive under symmetry.

3. *Cross*: this is the effect of an increase in one firm's cost on the equilibrium price of another firm $\rho_{ij} \equiv \frac{dp_j^\star}{dc_i}$. $\sigma_{ij} \equiv \frac{\rho_{ij}}{\rho_i^{eq}}$ is a reasonable measure of equilibrium strategic substitutability or complementarity (Fudenberg and Tirole, 1984; Bulow et al., 1985) in duopoly and extends to oligopoly naturally only under symmetry (Athey and Schmutzler, 2001), which is why we maintain that assumption (see our conclusion).

4. *Industry*: this is the effect of an increase in all firms' cost, on each firm's price $\rho_I = \frac{dp_i^\star}{dc}$.

[27]In particular, we assume that the Jacobian of $\mathbb{Q}$ at any price vector is Hicksian (Hicks, 1939; Gale and Nikaido, 1965). A grossly sufficient condition for this is Slutsky symmetry of demand, nearly always assumed for practical purposes in industrial organization applications by quasi-linear utility specifications or by the fact that the good is a small part of income when income effects are allowed (Berry et al., 1995). We also assume $\tilde{Q}$ (as a single-product demand function) satisfies DMR and SPAs and that the matrix of cross-partials of firm profits are globally Hicksian (equilibrium is unique). Our results continue to hold if this is only true locally and one considers comparative statics of the local equilibrium, but formally dealing with multiple equilibria and defining the local equilibrium here would be cumbersome.

Theorem 4. *Under the assumptions maintained in this subsection, $\rho_i - 1, \rho_i^{eq} - 1$ and $\rho_I - 1$ all have the same signs and these are the same (opposite) as the signs of ρ_{ij} if the goods are complements (substitutes). Therefore the entry of a new firm into the industry will reduce (increase) prices if there is cost absorption and substitutes (complements) or cost-amplificaiton and complements (substitutes). Similarly a merger (with no efficiencies) between two firms producing substitutes (complements) will raise (lower) the merging firm prices and raise the prices of the other firms under cost absorption but lower them under cost-amplificaiton. Analogous results hold for Nash-in-quantities (see appendix* Multiple Products *Section III).*

Proof. See appendix *Multiple Products* Sections I-II.

Intuitively the results follow from two observations. First the various pass-through rates are connected because when all prices rise, demand for each product must fall. Thus cross-effects on market power cannot dominate own-effects. Log-curvature therefore determines Sop, Lop and industry pass-through. Furthermore, by Bulow's reasoning (Subsection I.E) , an increase in the price of a substitute good, which thereby increases inverse demand for a product will lead to an increase in prices when pass-through; conversely for a complement.

This shows that the conventional wisdom (Eaton and Grossman, 1986; Rasmusen, 2006), shown by the case of $\rho < 1$ in Table 2, on the relationship between form of oligopoly and strategic effects is valid in this setting if and only if demand is cost-absorbing. The general falsity of this conventional wisdom (Fudenberg and Tirole, 1984; Bulow et al., 1985) created a damning ambiguity in the theory of oligopoly: one can obtain nearly any result or its opposite by the appropriate assumptions on demand. However, at least in this simple context, this ambiguity is resolved by the fact that observing either short or long-run effects of any one of a variety of cost shocks on even a single price identifies these qualitative effects. The entry of a new firm is effectively like the price of another firm falling from infinity and a merger like those prices rising (falling) in the case of substitutes (complements). Therefore pass-through rates allow us to predict the effect of competition of either kind on prices[28], a topic

[28] It is also simple to allow for the entry of a firm producing a complement when all other firms produce

$\rho < 1$	Substitutes	Complements
Bertrand	Strategic complements	Strategic substitutes
Cournot	Strategic substitutes	Strategic complements

$\rho > 1$	Substitutes	Complements
Bertrand	Strategic substitutes	Strategic complements
Cournot	Strategic complements	Strategic substitutes

Table 2: Strategic effects and pass-through rates with Horizontal Demand Systems

of recent theoretical interest (Chen and Riordan, 2008; Gabaix et al., 2009). Furthermore, Fudenberg and Tirole (1984) and Bulow et al. (1985), and a large literature they sparked, show that a wide range of issues in oligopoly theory turn on the distinction between strategic complements and substitutes, the ambiguity of which these results begin to resolve.

This reasoning can be made quantitative, implying a number of intuitive comparative statics on the effects of interaction strength (competition or complementing), pass-through rates and industry diffuseness on Sop, Lop, cross and industry pass-through. Let the strength of interaction $s \equiv \left|\sum_{j \neq i} \frac{\partial g}{\partial p_j}\right|$; under substitutes $s \leq 1$ as an increase in prices weakly reduces each firm's demand; for complements $s \leq N$ as complementary is at most perfect.

Theorem 5. *ρ_i^{eq} and ρ_I are increasing in ρ_i. ρ_i^{eq} is increasing in s and decreasing in N. ρ_I is constant in N and when goods are substitutes (complements) is increasing (decreasing) in s under cost-absorpton and decreasing (increasing) in s under cost amplification. σ_{ij} is decreasing (increasing) in ρ_i, so long as $\rho_i < 2$, and when goods are substitutes (complements), is increasing in absolute value in s and decreasing in absolute value in N.*

As N becomes large, ρ_i^{eq} approaches ρ_i and σ_{ij} approaches 0. Under substitutes as s approaches 1, ρ_i^{eq} approaches a finite limit $\tilde{\rho}_i^{eq} > \rho_i$ which is on the same side of 1 as ρ_i, σ_{ij} approaches some finite number of the appropriate sign and ρ_I approaches 1.

Proof. See appendix *Multiple Products* Sections I-II.

substitutes (a good complementary to any one of the competing products), or a substitute when all other firms produce complements (an alternative to the multimarginalized product), so long as the effects are symmetric. The results are intuitive: entry of a substitute or has the same qualitative effect as it would have if other goods had the same utility interaction.

Thus all the intuitions from Theorem 4 are quantitative as well as qualitative. The theorem provides, as far as we know, the first attempt to formally characterize the relationship between the degree of differentiation and pass-through rates in multi-product industries, a central topic in antitrust analysis (Farrell and Shapiro, 2008). Conventional wisdom (Besanko et al., 2001; Kim and Cotterill, 2008) is that strong competition leads to low Lop and industry pass-through near 1. Our theorem indicates that, with constant marginal cost, only the second of these intuitions is correct: increased differentiation *reduces* pass-through. Intuitively, competition increases strategic interactions[29]; because strategic effects are symmetric here, by the LeChatelier Principle (Samuelson, 1947), the stronger such effects are, the stronger the elasticity of reaction (pass-through) to changes in cost. However, in work in progress, Weyl shows that when marginal costs are not constant as assumed here, the relative elasticity of supply and demand (is the industry more competitive than it is close to constant marginal cost?) becomes crucial. If demand is much more elastic than supply, competition does drive down pass-through[30]. Nonetheless, Theorem 5 makes clear the delicacy of common intuitions about pass-through rates and establishes new ones regarding the relationship between pass-through rates and strategic effects[31].

B. Discrete choice models

While the results above provide basic intuitions and HDSs (especially linear demand) are often used in theoretical analysis, they are only occasionally applied empirically. Far more common are discrete choice models of demand such as the logit (McFadden, 1974; Werden and Froeb, 1994) and mixed logit models (Berry et al., 1995). These models are challenging to analyze theoretically and little is therefore known about their general implications for the comparative statics of oligopoly. Until recently the most comprehensive results were due to

[29]Increasing the number of firms, while holding s constant, decreases the strength of such strategic interactions by making them more diffuse along the lines of monopolistic competition, and thus has the opposite effect of increasing s.

[30]This sort of effect was first noted by Bishop (1968) in the context of monopoly.

[31]Preliminary research available on request indicates that these are robust to non-constant marginal cost.

Perloff and Salop (1985) who derive, for a few special distributions and under the assumption of independent consumer valuations across products, the effects of entry on prices.

Intuitively discrete choice models are closely tied to HDSs. Standard discrete choice models imply that the difference between the utility (and choice probabilities) of two goods depend on the prices of those goods only through the difference between the prices. However, given the complex choice probabilities arising from the discreteness of maximization and integration over consumer heterogeneity, the general validity of this intuition is unclear.

Luckily Gabaix et al. (2009) have recently made progress in understanding the oligopoly implications of fairly general discrete choice models with a large number of symmetric firms. They obtain analytic expressions for demand and its derivatives in the non-parametric Berry et al. (1995) (BLP) model, a generalization of mixed logit allowing arbitrary value distributions. It is straightforward to show that these formulae imply horizontality. This provides a tentative beginning of a bridge between the intuitions developed thus far and the comparative statics of common empirical demand models[32].

Furthermore, the log-curvature of idiosyncratic consumer variations of goods determines the log-curvature of demand in own price and therefore whether demand is cost-absorbing or cost-amplifying, there are strategic complements or substitutes, etc. Thus functional form assumptions that impose the log-curvature of idiosyncratic variations, which we show in Section V.A are often made, are strongly restrictive at least in this simple settings. This suggests, but certainly does not prove[33], that such assumptions strongly influence results in the more complex empirical models typically employed.

Theorem 6. *Theorems 4 and 5 apply to the Gabaix et al. (2009) non-parametric BLP model with a large number of symmetric firms. Furthermore in the Gabaix et al. (2009) model demand is cost-absorbing (amplifying) in the Sop sense near the symmetric equilibrium if the distribution of (idiosyncratic) valuations are log-concave (convex). If a distribution of*

[32]It also explains 6's main result on the effect of entry on prices. Their results apply to the comparatives statics of surplus in large auctions, another connection between auction theory and pass-through.

[33]Marginal restrictiveness of these assumptions may decline as a result of moving to a more complex model with asymmetries across firms. Consideration of this is an important topic for future research.

valuations has uniformly weakly lower (higher) pass-through rate than another, it induces in both models a weakly lower (higher) Sop.

Proof. See appendix *Multiple Products* Section IV.

IV. Other Applications

A. Two-sided markets

A popular recent topic in industrial organization has been"two-sided markets", industries with network effects that occur *between* two distinct groups of consumers. Typical examples are firms serving as a platform for transactions (payment cards), two-sided services (advertising, website access, video game playing) or matching (dating clubs or websites).

One influential model[34] was proposed by Rochet and Tirole (2003)(RT2003) and analyzed by Weyl (2009a). Consider a credit card company, call it Visa, which charges a per-transaction price to card-carrying consumers and card-accepting merchants. If exogenously the price Visa charges to merchants rises by a dollar, this acts as a subsidy dollar subsidy for consumers using their cards. Therefore the economics of the RT2003 model turn on the rate at which the firm finds it optimal to pass-through subsidies from one side to a reduction in prices to consumer on the other. Competition reduces the sum of prices on the two sides of the market when demand on both sides is cost-absorbing; if demand on one side is cost-amplifying, competition lowering prices on one side may raise prices overall.

On the normative side, firms only internalize the benefits that marginal consumers gain from more partners joining on the other side of the market (Spence, 1976; Weyl, Forthcoming), as they cannot price discriminate. The relative size of this surplus compared to the internalized mark-up are given by the pass-through rate according to the logic of Sec-

[34]The RT2003 model's multiplicative demand specification has been used as a building block in many other studies. For example, the main results of Hermalin and Katz (2004) turn on the distinction between cost absorption and cost amplification for the same reasons as in the RT2003. Unsurprisingly, models of platforms as competing groups of vertical monopolies (Carrillo and Tan, 2009) also largely turn on pass-through rates for reasons more closely related to our results in Section II.

tion I.D. Therefore, the overall price is a good gauge of welfare[35] when both demands are cost-absorbing and a poor gauge when one demand is cost-amplifying[36]. Cost shocks simultaneously allow for the distinction between these cases to be identified and the model to be tested.

Because the same properties of pass-through identify many simple models, more complicated ones need not be exponentially more complex (under-identified). In fact, Weyl (2008) provides an example of a case where the opposite happens, and the same assumptions allow exponentially more *simplicity* (testing). There I study a model combining the vertical and two-sided markets aspects of platforms to analyze vertical integration of intermediaries in two-sided markets, such as a merger between card-issuing banks and debit clearing networks or video game and joystick producers. Combining two problems which are identified and testable by SPAs leads to a model that is more easily testable than either of its components, offering hope that our approach may apply in more complex, realistic models.

B. Merger analysis

Static merger analysis in differentiated product industries typically turns on the evaluation of the anticompetitive effect of merging firms internalizing diverted profits and offsetting efficiencies (Williamson, 1968). Both of these shift costs faced by firms. Horizontal mergers tend to be anticompetitive as they increase the opportunity cost of sales faced by a firm, because after the merger it must take into account the lost profits cannibalized from the sale of a substitute product. Efficiencies, which may offset these anticompetitive effects, are reductions in firms' marginal costs as a result of productive synergies. As Froeb et al. (2005) and Farrell and Shapiro (2008) argue, with Nash-in-prices competition, once the relative size of efficiencies and cannibalization, and therefore the sign of the price effect (Werden, 1996),

[35]Another example of the importance of pass-through in this context is whether consumers would rather merchant prices rise or fall. Higher merchant prices will reduce prices to consumers, but also reduce the surplus earned by average consumers as they will have fewer merchants to visit. If consumer pass-through rates are increasing (decreasing) in price the second (first) effect dominates.

[36]Both demand being cost-amplifying violates second-order conditions.

of the merger is tied down, the magnitude is determined by the pass-through rate.

Thus shocks to the marginal costs of the merging firms alone[37], coupled with a measurement of efficiencies, identify[38] a local approximation of unilateral price effects: they yield estimates of the relevant pass-through rates (own and cross) and (nearly)[39] elastiticities. This provides a non-parametric (local) foundation for merger analysis that avoids rampant sensitivity of merger simulation to the functional form used, even given a collection of elasticities and cross-elasticities (Crooke et al., 1999). By construction, this ignores second-order effects, such as interactions between the anticompetitive effects on the two goods.

If one wishes to incorporate these effects[40] without restricting the pass-through rate which drives first-order effects, a natural approach is to formulate a tractable demand system which is known to allow flexible elasticities and pass-through rates[41]. The most straight-forward way to formulate such a demand system is to assume that (all) pass-through rates are constant in (all) prices and that demand is horizontal, the natural multi-product extension of the Bulow and Pfleiderer (1983) constant pass-through class of demand functions. In

[37]Obtaining data providing even such a limited number of cost shocks may be difficult, especially if the merging firms sell many products. However we follow Baker and Bresnahan (1985) in believing that reducing the estimation of properties of just two firms' demand systems constitutes a significant improvement. Unlike Baker and Bresnahan, our statement actually allows (locally approximations to) an estimate of merger effects, rather than merely the elasticities that, along with pass-through and efficiencies, determine it.

[38]For a more formal definition of the sense of identification here, see Weyl (2009b).

[39]This not quite correct. Such a shock does not literally identify the effects of changing each price on demand for other goods, because other firm prices change as well. Thus the diversion that a Nash-in-prices firm considers in its decision making is not quite estimated. However, the approximation is likely to be quite good for several reasons. First, if firms instead of playing a Bertrand game follow the generalized Bertrand analog of Bresnahan (1981)'s notion of consistent conjectures then the local approximation is precisely correct. Second, cross pass-through is second-order relative to Lop, especially in an industry with many firms, and thus the estimated elasticities likely give a close approximation to those that are relevant. Finally, because only the ratio of the cross-price demand slope to own-price demand slope, and not their levels, is relevant to measuring the diversion, only systematic differences in the effects of changes in other prices on these demands could bias calculations of cannibalization. We doubt such differences are have a first-order effect. However, formal investigation of these properties is an important area for future research.

[40]An alternative is to explicitly calculate the general determinants of second-order effects. While these might be hard to identify in practice, having a clearer sense of them would help guide other estimation approach and is therefore and interesting direction for future research.

[41]Of course this approach has the significant disadvantage of requiring estimates of *all* elasticities, cross-elasticiites and pass-through rates and not merely those of the merging firms. However techniques for measuring elasticities have advanced significantly in recent years and once a matrix of elasticities and cross-elasticities have been measured, a single cost shock to *any combination of goods* suffices to measure all pass-through rates in HCoPaDS. This property is, of course, a result of the restrictive horizontal nature of HCoPaDS and therefore comes at a cost of its own.

appendix *Constant Pass-through Demand Systems* Section I we show that these *Horizontal Constant Pass-through Demand Systems* (HCoPaDS)[42] take the form

$$Q_i(p_i, \mathbf{p}_{-i}) = \lambda_i \left([1-\rho_i] \left[\tilde{p}_i - p_i + \sum_{j \neq i} \beta_{ji} p_j \right] \right)^{\frac{\rho_i}{1-\rho_i}} \tag{11}$$

where $\lambda_i, \tilde{p}_i, \rho_i, \ \beta_{ji}$ are parameters that can be adjusted to set, respectively, the level of demand, the own-price elasticity, the Sop and cross elasticities. In addition to its flexibility, this demand system has the attractive[43] property, shown in appendix *Constant Pass-through Demand Systems* Section I, that solutions to oligopoly pricing are a simple linear matrix algebra problem $\mathbf{p}^\star = \mathbf{K}(\mathbf{c} + \alpha)$ where $\mathbf{c}$ is the vector of costs, $\mathbf{K}$ is a matrix of parameters determined by the values of ρ_i and β_{ji} and α the N-vector with typical entry $\alpha_i = \frac{\tilde{p}_i \rho_i}{1-\rho_i}$.

However these demand systems have one significant disadvantage. Except in the special linear case, they typically violate Slutsky symmetry; they are therefore difficult to rationalize from consumer preferences, limiting their usefulness in welfare analysis[44]. More generally CoPaDS can give strange answers outside of the local area in which they are calibrated and are therefore best thought of as extensions the local methods of Froeb et al. (2005) and Farrell and Shapiro (2008). These problems may be less severe than they at first appear, however, given that we are mostly interested in such local approximations (merger impacts are typically reasonably small) and to the extent that they are not the local data used to

[42] In that appendix we also describe a more general class of *Constant Pass-through Demand Systems*:

$$Q_i(p_i, \mathbf{p}_{-i}) = f^i(\mathbf{p}_{-i}) \left([1-\rho_i] \left[\tilde{p}_i - p_i + \sum_{j \neq i} \beta_{ji} p_j \right] \right)^{\frac{\rho_i}{1-\rho_i}} \tag{10}$$

where a smooth, positive function f_i is used to obtain arbitrary levels and cross-price elasticities as a function of other firm prices given own-price. These retain the advantages of HCoPaDS, but lose horizontality.

[43] For example, this implies analytic global comparative statics of prices with respect to all parameters, making sensitivity analysis of any result highly transparent, a unique solution (so long as $\mathbf{K}$ is non-singular) and makes the imposition of stability conditions easy through the restriction of $\mathbf{K}$. It has the additional useful feature that a post-merger equilibrium when two firms merge is well-approximated by the solution to a low-order polynomial equation as long as no ρ_i is close to 0 or 1 (see appendix *Constant Pass-through Demand Systems* Section II).

[44] We are currently working to formulate a Slutsky symmetric flexible demand system that overcomes this difficult, at the cost, of course, of the tractability of CoPaDS.

estimate these effects are likely to be insufficient[45]. Local Slutsky symmetry can easily be imposed so for small changes in prices, using the Harberger approximation that consumer welfare loss from a change in prices is $\mathbf{Q}(d\mathbf{p})^\top$ may still be quite accurate, even if the demand system does not globally obey the assumptions justifying this approximation.

Nonetheless because of its tractability, HCoPaDS allows the investigation of the demand properties needed to support typical assumptions used in applied policy analysis. We provide one example here. Farrell and Shapiro (2008) assume that if a merger creates a local incentive for price increases on both products (*Upward Pricing Pressure* [UPP] in their terminology), both products' prices will rise in equilibrium. Clearly this holds if the two prices are complements (supermodular) for the firm and prices across firms are strategic complements or if the two good are symmetric. However many have argued that prices are likely substitutes across goods within a firm (Hausman, 1997), as higher prices reduce market shares and therefore cannibalization across products. We are not aware of any conditions on demand sufficient for the assumption to be valid. We provide an example in the HCoPaDS class where this fails (even for the local optimum) and then offer fairly restrictive assumptions about demand within HCoPaDS ensuring that such a *strictly anticompetitive merger* raises both prices. We focus on the case of two firms merging to monopoly, allowing us to focus on the failure of complementarity within the merged firm.

Example 1. *Consider the limit case when $\rho_i \to 1$ and demand is exponential. The goods are symmetric pre-merger with constant marginal cost $c_i = 1$ and demand $Q^i = e^{.2p_j - p_i}$; Slutsky symmetry is thus locally satisfied. Pre-merger optimal prices of the firms are $p_i^\star = 2$. The merger creates efficiencies in the pricing of good 1, reducing its marginal cost of production to $\tilde{c}_1 = .81$ but not in the production of good 2 whose marginal cost remains at 1. Then, by Farrell and Shapiro (2008)'s formula the UPP is the net of diversion and efficiencies:*

$$UPP_1 = -\frac{Q_1^2}{Q_1^1}(p_2 - c_2) + \tilde{c}_1 - c_1 = .2\frac{e^{.2p_i - p_j}}{e^{.2p_j - p_i}}1 - .19 = .01 > 0$$

[45] See our discussion in Subsection I.B above for more details.

$$UPP_2 = .2 \cdot 1.19 = .238 > 0$$

Thus the merger is strictly anticompetitive. But post-merger first-order conditions are

$$\tilde{p}_1^\star = 1.81 + .2(\tilde{p}_2^\star - 1)e^{1.2(\tilde{p}_1^\star - \tilde{p}_2^\star)}$$

$$\tilde{p}_2^\star = 2 + .2(\tilde{p}_1^\star - .81)e^{1.2(\tilde{p}_2^\star - \tilde{p}_1^\star)}$$

Again, HCoPaDS can give implausible predictions outside a local range of prices. We therefore restrict post-merger prices for the goods to be in the range $[1, 2.5]x[1, 2.5]$. *Over this range, profits are concave as shown in the appendix Section III, so the the solution to the above first-order conditions is the unique (interior) local optimum. Solving numerically, post-merger optimal prices are* $\tilde{p}_2^\star = 2.38$ *but* $\tilde{p}_1^\star = 1.98 < 2$. *Thus prices fall for one good, though a local approximation indicates consumer welfare falls by* $.36 \cdot e^{-1.6} = \$.07$.

Despite price changes being fairly large (20% in the case of good 2), the Farrell and Shapiro (2008) local approximation performs reasonably well, predicting $p_2^\star = 2.24$ and $p_1^\star = 2.01$ especially when compared to the $p_2^\star = 2.12$ and $p_1^\star = 2$ that would have obtained from linear demand, though it clearly understates the welfare harms.

Proposition 1. *A strictly anticompetitive merger-to-monopoly between two firms with horizontal demand that*

1. *is weakly concave in its own price* ($\rho_i \leq \frac{1}{2}$ *for both* i),
2. *has both diversion ratios* $-\frac{\frac{dQ^j}{dp_i}}{\frac{dQ^i}{dp_i}}$ *less than* 1
3. *and has the property that consumption of both goods falls when both prices rise*

leads to higher equilibrium prices for both products.

Proof. See appendix *Constant Pass-through Demand Systems* Section III.

Thus if pass-through is sufficiently low for both products, a strictly anticompetitive merger increases both prices, though the magnitude of the effects are small.

C. Third-degree monopoly price discrimination

A very recent paper (Aguirre et al., Forthcoming) provides a particularly elegant application of pass-through. The authors return to one of the oldest questions in industrial organization, posed by Pigou (1920): when does explicit third-degree price discrimination by a monopolist raise output and/or welfare? This question has been studied over the years by Robinson (1933), Schmalensee (1981) and Varian (1985), among others. Aguirre, Cowan and Vickers (ACV) prove simple results which immediately generalize all of these.

Consider two markets, high (H) and low (L). Absent discrimination, prices are constrained to be identical. With discrimination, prices in H exceed those in L by an Δ. ACV propose a natural continuous path from no discrimination to discrimination: we require that $p^H < p^L + \delta$. Assume profits in each market π^H and π^L are concave in price. Then for any $\delta \in [0, \Delta]$, the monopolist will choose $p^H = p^L + \delta$. Her first-order condition is thus $\pi^{H'}(p^L + \delta) + \pi^{L'}(p^L) = 0$. For $\delta < \Delta$, $\pi^{L'} < 0 < \pi^{H'}$, but these both converge to 0 as δ goes to Δ.

A firm facing exogenous quantity (see Subsection I.C) $\tilde{q}$ earns profits $(Q[p] - \tilde{q})(p - c)$. Her first-order condition is thus $Q'(p)(p-c) + Q(p) - \tilde{q}$, while the first-order condition in the high market in the price discrimination problem are $Q^{H'}(p)(p-c) + Q^H(p) + \pi^{L'}(p - \delta)$. In effect, the downward pressure on prices from the constraint against discrimination in the low market enters exactly like exogenous quantity. Moving towards discrimination is therefore equivalent to moving exogenous quantity from the high market to the low market.

Thus ACV show that discrimination leads to higher output if an average of quantity pass-through in the low market exceeds that in the high market. Similarly the change in social welfare in each market from the change in quantity is mdq so a comparison of an average of the mark-up times the quantity pass-through over the relevant range in the two markets determines the welfare effect of discrimination. The connections of pass-through to demand curvature make it clear how this result immediately implies the famous prior results of Pigou, Robinson, Schmalensee and Varian on the connections between demand curvature and the effects of discrimination.

D. Price controls and consumer welfare

Bulow and Klemperer (2009) provide an application to the regulation of competitive markets. Consider a competitive market with perfectly inelastic supply. A price control benefits consumers by offering them lower prices, leading them to gain the marginal surplus, i.e. the difference between price and marginal revenue μ. However if the rationing, made necessary by the price control, randomly allocates the good among consumers, the good is reallocated from a consumer earning the average surplus $\frac{V}{Q}$ to a marginal consumer earning no surplus; thus consumers lose $\frac{V}{Q}$. Which of these is larger is, by Theorem 3, determined by the average pass-through at higher prices. For example, under cost absorption consumers are better off with the control, while under cost amplification they are better off without it.

E. Auctions

In a series of recent papers[46], Mares and Swinkels use an independent derivation of the corollaries of Theorem 3 to characterize when first- or second-price mechanisms are revenue-preferred as asymmetric auctions. Their analysis is almost entirely in terms of what we have referred to as pass-through rates, their slope and the ratio of consumer to producer surplus. However, the formal connection between their analysis and the reasoning here is not immediately clear, as they use a more traditional auction-theoretic approach in the sprit of Myerson (1981) rather than the price theory approach of Bulow and Roberts (1989). Still, we suspect, given the duality of auctions and price discrimination that Bulow and Roberts established, that the connection is quite tight. We are therefore optimistic, based both on the Mares-Swinkels results and those of Gabaix et. al. and ACV, that pass-through will prove fruitful in theory of auctions more broadly.

[46]The first of these is publicly available and cited in bibliography and the rest are currently being prepared by the authors, likely to be available by December.

V. Functional Forms and Pass-through

We have shown that the distinction between cost absorption and cost amplification, as well as between increasing and decreasing pass-through, organizes the comparative statics of many industrial organization models. Because we have little solid basis for generally determining which case holds (Subsection I.F), it is important that functional forms used for both theoretical and empirical analysis are flexible along these dimensions. We argue that currently-available functional forms fail this test and offer a new functional form, *Adjustable pass-through* (Apt) demand, solving this problem while maintaining tractability.

A. A taxonomy of functional forms

Common functional forms in industrial organization fall into three categories:

1. Most common in theoretical work and empirical analysis of single-product industries are the Bulow and Pfleiderer (1983) *constant pass-through* class of demand functions, which include linear (uniform), constant elasticity (Pareto) and constant mark-up (negative exponential) demands as special cases. This class is highly tractable, yielding linear solutions to monopoly problems, and allows flexible pass-through[47]. However, it is defined by constant pass-through and therefore imposes the slope of pass-through.

2. A common class of demand functions are those based on statistical distributions. These are more often used as building blocks for multi-product demand systems (McFadden, 1974; Berry et al., 1995) than as direct demand functions, though the log-curvature properties of these are often connected (Subsection III.C).

3. The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) with constant expenditures has been used in many applications (Hausman, 1997).

[47]If, unlike in most empirical analysis, the pass-through is actually estimated rather than assumed.

	$\rho < 1$	$\rho > 1$	Price-dependent	Parameter-dependent
ρ' $\wedge$ 0			AIDS with $b < 0$	
ρ' $\vee$ 0	Normal (Gaussian) Logistic Type we Extreme Value (Gumbel) Laplace Type III Extreme Value (Reverse Weibull) Weibull with shape $\alpha > 1$ Gamma with shape $\alpha > 1$		Type II Extreme Value (Fréchet) with shape $\alpha > 1$	
Price-dependent				
Parameter-dependent				
Does not globally satisfy DMR		Type II Extreme Value (Fréchet) with shape $\alpha < 1$ Weibull with shape $\alpha < 1$ Gamma with shape $\alpha < 1$		

Table 3: A taxonomy of some common demand functions

The pass-through properties of the first class are immediate. Table 3 provides a taxonomy, established formally in appendix *Taxonomy of Functional Forms*, of the pass-through properties of the second class[48] and a single-product AIDS described in the appendix.

Of course, there is no reason why a particular class should fall into one of the four categories permitted by the SPAs: different parameter values and/or prices might well lead to different pass-through rates and slopes. Table 3 allows for violations of SPAs. However, strikingly many commonly used distributions *do* turn out to be simply classifiable according to this taxonomy. This perhaps provides a vague justification for the SPAs we use. More persuasively they show how in problems where the level and slope of pass-through are crucial, many commonly-used demand functions are implausibly restrictive, at least in the single-product case. If the distinction between cost absorption and cost amplification (or between increasing and decreasing pass-through) determines the effects of a policy, then the assumption that demand is of almost any of the common forms[49] is not an innocent simpli-

[48]The reader should understand by a probability distribution F a demand function $D(p) = A\big(1 - F[p]\big)$; because pass-through is scale-invariant, all categorization hold for arbitrary positive A.

[49]With the exception of AIDS. And even this is price dependent (neither is globally cost-amplifying). That is while AIDS *does* have pass-through rates that can be either cost-absorbing or cost-amplifying it *does not* allow these to be flexible once the elasticity and level of demand have been tied down. That is, while it is first-order flexible it is not flexible on pass-through given these first order properties. In this way it suffers from the same defects as, say, constant elasticity demand (Bulow and Pfleiderer, 1983).

Name	Parameter values	Demand form
Limiting cost absorption	$\overline{\rho} < 1$	$\lambda \left([1-\overline{\rho}]\sqrt{\overline{p}-p} - 2\overline{\rho}\alpha \right)^{\frac{2\overline{\rho}}{1-\overline{\rho}}}$ $p < \overline{p} - \frac{4\alpha^2\overline{\rho}^2}{(1-\overline{\rho})^2} 1_{\alpha>0}$ 0 $\overline{p} - \frac{4\alpha^2\overline{\rho}^2}{(1-\overline{\rho})^2} 1_{\alpha>0} \leq p$
Cost-absorbing constant limiting mark-up	$\overline{\rho} = 1, \alpha < 0$	$\lambda e^{-\frac{\sqrt{\overline{p}-p}}{\alpha}}$ $p < \overline{p}$ 0 $\overline{p} \leq p$
Constant mark-up	$\overline{\rho} = 1, \alpha = 0, \mu > 0$	$\lambda e^{-\frac{p}{\mu}}$
Cost-amplifying constant limiting mark-up	$\overline{\rho} = 1, \alpha < 0$	∞ $p \leq \underline{p} - \alpha^2$ $\frac{2\lambda\alpha^2}{e(p-\underline{p}+\alpha^2)}$ $\underline{p} - \alpha^2 < p \leq \underline{p} + \alpha^2$ $\lambda e^{\frac{\sqrt{p-\underline{p}}}{\alpha}}$ $\underline{p} + \alpha^2 < p$
Limiting cost amplification	$\overline{\rho} > 1$	∞ $p \leq \underline{p} + \alpha^2\overline{\rho} \left(\frac{4\overline{\rho}}{(1-\overline{\rho})^2} 1_{\alpha>0} - 1_{\alpha<0} \right)$ $\lambda \left(\overline{\rho}[\overline{\rho}+1] \right)^{-\frac{\overline{\rho}+1}{\overline{\rho}-1}} (\alpha^2)^{-\frac{1}{\overline{\rho}-1}} \left(p - \underline{p} + \alpha^2\overline{\rho} \right)^{-1}$ $\underline{p} - \overline{\rho}\alpha^2 1_{\alpha<0} < p \leq \underline{p} + \overline{\rho}^2\alpha^2 1_{\alpha<0}$ $\lambda \left([\overline{\rho}-1]\sqrt{p-\underline{p}} - 2\overline{\rho}\alpha \right)^{-\frac{2\overline{\rho}}{\overline{\rho}-1}}$ $\underline{p} + (\overline{\rho}\alpha)^2 \left(1_{\alpha<0} + \frac{4}{(1-\overline{\rho})^2} 1_{\alpha>0} \right) < p$

Table 4: The forms of Apt demand $Q(p)$ for various parameter values, all with $\overline{\rho}, \lambda > 0$

fying assumption for computational purposes or even a questionable structural restriction. Instead it drives the analysis of an "empirically estimated" model, or the analytical conclusion of a theoretical analysis, entirely independent of the data or the intuitive economic environment.

B. Adjustable-pass-through (Apt) demand

These restrictions can be eliminated, without sacrificing tractability, by generalizing the constant pass-through demand class to allow flexibility in the slope as well as level of pass-through. This leads naturally to a form for demand that we call *Adjustable-pass-through* (Apt). Apt demand takes a few different forms, depending on the parameter values, which are shown in Table 4. These parallel the forms of the constant pass-through class, which may be transformed-linear or translated constant elasticity depending on pass-through rates, which it generalizes. Apt demand has a number of other attractive properties which we state and establish formally in appendix *Apt Demand*. Apt demand

1. nests as special cases all previous demand forms for which the monopoly problem can be explicitly solved (constant pass-through class);

2. is weakly positive, monotone decreasing and smooth almost everywhere that matters;

3. satisfies DMR and gives quadratic solutions to monopoly pricing for all cost levels;
4. satisfies SPAs, including in the GCS extensions discussed in Subsection II.B;
5. can match arbitrary combinations of levels, elasticities, pass-through rates and a wide range of slopes thereof, making it more flexible than any common demand form;
6. is easy to estimate based on a second-order regression of prices and quantities on cost shocks, or a third order regression of quantities on prices;
7. has a simple closed form consumer surplus and thus can easily be derived from the utility maximization of a representative consumer;
8. always gives simple, explicit solutions for final price and any mark-up in the GCS vertical monopolies model, leaving flexible the relevant pass-through rates and slopes;
9. can therefore be used to compare, in a mechanical yet quite general way, a wide range of equilibrium outcomes across and within industrial organizations.

While Apt demand is convenient for monopoly pricing and Bertrand games, it is less tractable for production decisions and Cournot games. We therefore propose another demand form, *Apt inverse demand*, where mark-up as a function of quantity takes the Apt form. Apt demand can also form the basis of a class of statistical distribution by interpreting the demand as the survivor function of a distribution of valuations[50]; as far as we know this is the first class of statistical distributions allowing flexibility in the first two derivatives of inverse hazard rates[51]. A computational toolkit accompanying this paper[52] allows researchers to easily estimate, manipulate and predict using Apt (inverse) demand.

[50] It is therefore easy also to derive Apt demand from a statistical distribution of consumer valuations.

[51] We hope this may be of some use in other fields, such reliability theory (Barlow and Proschan, 1975) and statistics (Cox, 1972) where hazard rates play an important role.

[52] Designed by our outstanding research assistant Yali Miao.

VI. Conclusion

This paper argues that primitive properties of demand which determine the pass-through rate a monopolist would choose play an important role in a wide range of industrial organization models. We demonstrate that reformulating the comparative statics of these models in terms of pass-through rates makes apparent connections between many predictions, increasing their empirical content under weak assumptions. Finally we have shown that the lack of understanding of the role played by pass-through has lead theoretical and empirical analysis to impose functional forms that restrict pass-through rates in implausible ways and have proposed a tractable classes of demand functions and systems avoiding these restrictions.

Nonetheless, our results are directly empirically relevant only in applications where simple models are plausible. For many applications our results leave out important features. We are actively working to extend our approach to address these shortcomings. Extensions in progress include allowing for imperfect supply elasticities, mutliproduct firms, asymmetries in multi-product industries, vertical product and consumer differentiation and discrete choice models with arbitrary numbers of firms. Other promising extensions are incorporation of information on product characteristics and income distributions, discrete choice demand systems employing pass-through-flexible random coefficient and idiosyncratic variation distributions, and non-parametric approaches to demand estimation based on the most relevant properties of demand (first elasticity, then pass-through, then its slope, etc.).

Another important direction for future research is the investigation of the relevance of results here to contexts beyond static oligopoly. A few classic topics in industrial organization seem clearly related. Demand-side incentives for collusion depend on the relative value of the gains to monopoly and the incentives for defection, which are independent of elasticities (Ivaldi et al., 2003) but seem clearly tied to pass-through rates or their slope. Given the connection between dynamic oligopoly investments, Stackelberg competition and strategic effects established by Fudenberg and Tirole (1984) and Bulow et al. (1985), pass-through rates are likely to prove useful in analyzing at least some models of dynamic oligopoly. Given the results of Mares and Swinkels and ACV, the connection of pass-through to auction theory and implicit (second-degree) price discrimination seems particularly promising. Further afield, pass-through may be connected to optimal tax design, given the connections between optimal income taxation and optimal price discrimination and the important role Saez (2001) shows log-curvature plays in optimal tax theory[53]. Recent work by Gopinath and Itskhoki (Forthcoming) shows that the primitives used here also link exchange rate pass-through to the frequency of firms' price adjustments, suggesting a connection to international economics.

We hope future research will relax the assumptions underlying the results derived here. The SPAs proposed in Subsection I.E are certainly much stronger than needed for most results and obscure natural quantitative information: if pass-through is significantly, rather than slightly, below unity, this should clearly count for something. More careful statistical formulation of assumptions for particular results would therefore be helpful for empirical applications. Furthermore, because the results here help remove some of the ancillary assumptions typically used for identifying industrial organization models, they help expose the maintained economic assumptions to falsification. This holds out hope of confirming underlying economic theory, rejecting it in favor of alternative models or relaxing assumptions needed for identification, such as knowledge by the firm of the true demand system.

Finally, if our results were ever to be used in policy applications this would clearly provide firms with incentives to distort their pass-through rates. Such a critique in the spirit of Lucas (1976) applies to many standard approaches in empirical industrial organization. We hope the transparency of the approach here might eventually allow for the formulation of policy-oriented empirical approaches that take into account such firm incentives.

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