

HOW TO THEORIZE ABOUT A PRIORI KNOWLEDGE: A CASE STUDY

Albert Casullo

There are four distinct approaches to analyzing the concept of a priori knowledge. The four approaches can be distinguished by posing two questions:

1. What is the primary target of the analysis?
2. Does the analysis of the primary target presuppose some general theory of knowledge or justification?¹

There are two primary targets of analysis. A reductive approach analyzes the concept of a priori knowledge in terms of the concept of a priori justification: S knows a priori that p just in case S's belief that p is justified a priori and the other conditions on knowledge are satisfied. The primary target of analysis is the concept of a priori justification.² A nonreductive approach offers an analysis of the concept of a priori knowledge in terms of conditions that do not involve the concept of the a priori. The primary target of analysis is the concept of a priori knowledge.

There are two approaches to analyzing the primary target. A theory-neutral approach provides an analysis of the primary target that does not presuppose any general theory of knowledge or justification. A theory-laden approach provides an analysis that does presuppose some general theory of knowledge or justification (call it *the background theory*).³

Those who embrace a theory-laden analysis incur a special burden: they must separate features of their analysis that are constitutive of the a priori from those that are constitutive of the background theory. The features that are constitutive of a priori knowledge are those that *differentiate* it from a posteriori knowledge. The features that are constitutive of the background theory are those that must be satisfied in order for any belief to count as knowledge or justified;

they are *common* to both a priori and posteriori knowledge. My goal is to illustrate, by means of a case study, how the failure to separate these features can lead to erroneous conclusions about the nature of a priori knowledge.

Philip Kitcher (1983), following W. V. Quine (1963) and Hilary Putnam (1983), offers the following influential argument in support of the conclusion that mathematical knowledge is not a priori: (1) the concept of a priori knowledge entails that a priori warrant is indefeasible by experience; but (2) the warrant conferred by the sources of mathematical knowledge touted by proponents of the a priori is defeasible by experience.⁴ Most of Kitcher's critics, including myself, reject (1).⁵ In a recent paper, Kitcher (2000) addresses this criticism. There are two divergent strands to his response. The first concedes that his original supporting argument for (1) is flawed, but offers a new multi-faceted defense. The second contends that the important question about mathematical knowledge is not whether it is a priori but whether it is tradition-independent.

I defend three theses in this paper. First, Kitcher's analysis of a priori knowledge is reductive and theory-laden, but his defense of (1) is undercut by a failure to distinguish between the requirements of the background theories that he discusses—Cartesian foundationalism, reliabilism, and socio-historicism—and those constitutive of the a priori. Second, Kitcher's contention that the important question about mathematical knowledge is whether it is tradition-independent is also undercut by his failure to distinguish between the requirements of the socio-historical theory of knowledge that he favors and the requirements of the a priori. Third, once we distinguish between the requirements of the background theories of knowledge that Kitcher discusses and those of the a priori, we can see that the traditional question of whether

mathematical knowledge is a priori remains central to the current debate over the nature of mathematical knowledge.

I

Kitcher (2000, 66) offers a theory-laden analysis of the concept of a priori knowledge:

The account of a priori knowledge is embedded within a general approach to knowledge (the *psychologistic* approach) according to which whether or not a state of true belief counts as a state of knowledge depends on the causal process that generated that state. If a state is produced by the right kind of causal process, so that it is a state of knowledge, then I say that the process is a *warrant* for the belief. My general understanding of warrants is a version of reliabilism . . . warrants are processes belonging to types that regularly and reliably produce true belief.

Applying the background theory to the a priori, Kitcher (2000, 67) maintains that

X knows a priori that p iff X knows that p and X 's knowledge that p was produced by a process that is an a priori warrant for p .

α is an a priori warrant for X 's belief that p just in case α is a process such that for any sequence of experiences sufficiently rich for X for p

- (a) some process of the same type could produce in X a belief that p ;
- (b) if a process of the same type were to produce in X a belief that p , then it would warrant X in believing that p ;
- (c) if a process of the same type were to produce in X a belief that p , then p . (67)

A sequence of experiences sufficiently rich for X for p is one that is sufficient to acquire the concepts necessary to entertain p .

Since Kitcher's account is theory-laden, it is important to understand the requirements of the background theory. Suppose that S's true belief that p is produced or sustained by a process Φ . Goldman maintains that, for S to know that p, three conditions must be satisfied.⁶ First, Φ must be *locally* reliable: there is no relevant alternative situation in which Φ produces the belief that p and p is false. Second, Φ must be *globally* reliable: it has a tendency to produce a sufficiently high proportion of true beliefs on all actual, and some possible, occasions of use. Third, S's belief that p must be justified, which requires that (a) it is permitted by a right system of J-rules, i.e., one that permits basic psychological processes whose instantiation would result in a sufficiently high truth ratio of beliefs, and (b) this permission is not undermined by S's cognitive state.⁷

Kitcher's analysis is of the concept of a priori knowledge is theory-laden. Is it also reductive? There are two reasons for viewing his analysis as reductive. First, Kitcher analyzes the concept of a priori knowledge in terms of the concept of a priori warrant and he (1983, 25) states that his notion of warrant is the same as Goldman's notion of justification.⁸ Second, Goldman (1986, 53) maintains that justification is necessary for knowledge because of the following consideration:

There are cases in which beliefs are caused by globally and locally reliable processes, yet the person has reason to believe they are not reliable. These are cases where the belief, though caused by a reliable process, is not *justified*.

In such cases, the subject fails to know because his justification is undermined by his evidence that the belief forming process in question is not reliable. Condition (b), which is the distinctive condition of Kitcher's analysis, involves the concept of warrant. The examples that he employs

to show that this condition is not satisfied are of the same type as those that Goldman utilizes to show that the justification condition is not satisfied. They are cases of reliably produced belief accompanied by experiential evidence of unreliability. According to the background theory that Kitcher endorses, such experiences block knowledge by undermining justification.

Condition (a) of Kitcher's analysis is intended to capture the idea that S's justification for the belief that p is a priori only if it is available independently of experience (apart from the experiences necessary to acquire the concepts that are constituents of p). Although some critics question whether Kitcher's formulation adequately captures that idea, none maintains that the idea itself should be rejected.⁹ Those who reject Kitcher's analysis focus on conditions (b) and (c). Kitcher's conditions (b) and (c) share a common feature: they impose higher standards on a priori justification than those required by the background theory that he endorses. I (1988, 2003) have argued that, in the absence of some compelling argument in support of them, the higher standards are *ad hoc* and should be rejected.

The problem is clear in the case of condition (c), which precludes the possibility of a priori justified false beliefs. Reliabilism does not preclude the possibility of justified false beliefs: perceptual processes, for example, can justify false beliefs provided that they are generally reliable. So what is the basis for the higher standard on a priori justification? Kitcher (2000, 72) maintains that (c) is a consequence of (b). Thus, the burden of supporting the higher standards falls entirely on supporting (b); and, moreover, supporting (b) independently of (c).

Condition (b) also imposes a higher standard on a priori justification than that entailed by reliabilism. It requires that S's a priori justified belief that p be indefeasible by experience in any world in which S has sufficient experience to acquire the concepts in p. Yet perceptual processes

can justify S's belief that p despite the possibility of empirical defeating evidence. Why should a priori justification be treated differently?

Kitcher (1983, 89) originally argued that the higher standard is supported by the intuitive idea that a priori knowledge is independent of experience:

But if alternative experiences could undermine one's knowledge then there are features of one's current experience which are relevant to the knowledge, namely those features whose *absence* would change the current experience into the subversive experience. The idea of the support lent by kindly experience is the obverse of the idea of the defeat brought by uncooperative experience.

My (1988, 220-221) original argument against Kitcher questioned his account of the relationship between supporting and defeating evidence. It is uncontroversial that if S's belief that p is supported (i.e., justified) by experience then S's belief that p is not justified (and, hence, not known) a priori. But suppose that S's belief that p is justified nonexperientially and that S's nonexperiential justification for the belief that p is defeasible by experience. From the fact that S's justification for the belief that p is defeasible by experience, it does not follow that S's belief that p is supported (i.e., justified) by experience unless one presupposes the following symmetry between supporting evidence and defeating evidence:

(ST) If evidence of kind A can defeat the warrant conferred on S's belief that p by evidence of kind B, then S's belief that p is based on evidence of kind A.

But (ST) is not very plausible. For example, suppose that my present belief that there is a canary in the backyard is justified by my present perceptual experiences. Moreover, suppose that a friend who is an expert bird watcher also sees the bird, recognizes that it is not a canary but a

goldfinch, but to spare me embarrassment does not tell me that it is not a canary. Although the testimony of my friend would defeat my perceptual justification for the belief that the bird is a canary, it does not follow that my belief is justified by testimony. According to Kitcher (2000, 73), Charles Parsons makes a similar point: “the fact that under imaginable circumstances I could have grounds for believing that my perceptual experience isn’t veridical doesn’t entail that the absence of those experiences are now playing a causal role in generating or sustaining my perceptual beliefs.”¹⁰ Kitcher now concedes this point and agrees that his original defense of (b) fails.

II

Kitcher’s new response to his critics consists of two parts: (1) he argues that the rival conception of the a priori favored by his critics has significant shortcomings; and (2) he contends that the important question about mathematical knowledge is not whether it is a priori but whether it is tradition-independent. I follow Kitcher in referring to his conception, which includes both conditions (a) and (b), as the *Strong conception* or (SC); and the rival conception, which includes only (a), as the *Weak conception* or (WC). Let us introduce the term ‘nonexperiential process’ to refer to processes that are available independently of experience. We can now articulate the Strong and Weak conceptions, respectively, as follows:

- (SC) S’s belief that p is justified a priori iff S’s belief that p is justified by a nonexperiential process and that justification cannot be defeated by experience.
- (WC) S’s belief that p is justified a priori iff S’s belief that p is justified by a nonexperiential process.

There is a further complication that must be addressed before turning to Kitcher's arguments against (WC).

Kitcher no longer endorses the background theory of knowledge that informed his original defense of (SC). He now rejects reliabilism in favor of a socio-historical conception of knowledge. As a result, his arguments against (WC) fall into two categories: (a) those that presuppose reliabilism; and (b) those that presuppose socio-historicism. Since Kitcher's contention that the most important question about mathematical knowledge is whether it is tradition-independent also derives from his commitment to socio-historicism, I will address that contention and his arguments against (WC) that presuppose socio-historicism in section III. I address his arguments against (WC) that presuppose reliabilism in this section.

Kitcher offers three sets of considerations against (WC) that presuppose reliabilism. The first alleges that (WC) is satisfiable only if (SC) is satisfiable. The second contends that (WC) fails to capture an important feature of the traditional conception of the a priori. The third maintains that (WC) is too weak: there are cases of knowledge that appear to be empirical but satisfy (WC).

IIa

In support of the contention that (WC) is satisfiable only if (SC) is satisfiable, Kitcher (2000, 74) invites us to "envisage Gauss or Dedekind or Cantor coming to a priori knowledge that nobody has had before on the basis of some kind of process (call it 'intuition')." Let us assume that intuition is sufficiently reliable to produce justified beliefs but that there are possible experiences that call into question the reliability of intuition. Here it appears that a proponent of (WC) is in a position to argue that since these undermining experiences are not present in the

actual situation, mathematical beliefs formed on the basis of intuition are justified a priori, whereas a proponent of (SC) must concede that such beliefs are not justified a priori.

Kitcher, however, maintains that this difference between the two conceptions is only apparent. In support of this claim, he cites three considerations: (C1) the ability of mathematicians such as Cantor to attain knowledge of new mathematical principles is rare; (C2) since it is rare, it is difficult to find others who can verify that it has been exercised appropriately; and (C3) the history of mathematics indicates that the exercise of this ability has had variable results. Kitcher (2000, 75) concludes that “appeals to elusive processes of a priori reason ought always to be accompanied by doubts about whether one has carried out the process correctly, and whether, in this instance, the deliverances are true.” The upshot of this conclusion is that “The power [of intuition] to warrant belief in the actual situation would be undermined—and, indeed, we might claim that one couldn’t satisfy the Weak conception unless the Strong conception were also satisfied” (Kitcher 2000, 75).

Kitcher’s argument raises two questions. First, is his argument sound? Does it establish that (WC) is satisfiable only if (SC) is also satisfiable? Second, if his argument is sound, what does it show about the concept of a priori knowledge? Does it provide a basis for preferring (SC) over (WC)?

Kitcher contends that Cantor’s mathematical beliefs, although reliably produced by the process of intuition, are unjustified. They are unjustified because the justification conferred on them by the process of intuition is undermined by the fact that

- (D) Cantor ought always to have doubts about whether he has exercised the process correctly and whether the resulting beliefs are true.

Let us grant that if Cantor's mathematical beliefs are accompanied by such doubts then his justification for those beliefs is undermined. But why should we suppose that his exercise of the process of intuition ought always to be accompanied by such doubts?

Kitcher maintains that (D) is a consequence of (C1)-(C3). But (C1)-(C3) fail to support (D) since it need not be the case either that Cantor believes (C1)-(C3) or that his cognitive state justifies him in believing (C1)-(C3). But if he does not believe (C1)-(C3) and his cognitive state does not justify him in believing (C1)-(C3), then it is hard to see on what basis Kitcher can sustain the claim that Cantor's exercise of the process of intuition ought to be accompanied by such doubt.¹¹ Moreover, the contention that the justification conferred on Cantor's mathematical beliefs by the process of intuition is undermined by (D) is not supported by Kitcher's background theory. Goldman maintains that whether one's justification is undermined is a function of one's cognitive state. More specifically, the doubts in (D) undermine one's justification only if either one has such doubts or one's cognitive state justifies having such doubts. Since reliabilism does not require that Cantor have such doubts or that his cognitive state justifies having such doubts, it follows that the justification conferred on his mathematical beliefs by the process of intuition need not be undermined. Therefore, it is possible that Cantor's mathematical beliefs satisfy (WC) but not (SC).

But suppose that Kitcher's argument is cogent. Does it provide any basis for favoring (SC) over (WC)? No. Kitcher introduced (SC) in order to offer the following argument against mathematical apriorism: (1) (SC) is the correct analysis of the concept of a priori justification; but (2) the justification conferred on mathematical beliefs by the processes touted by proponents of the a priori is defeasible by experience. But if it is true, as he contends, that (WC) is

satisfiable only if (SC) is satisfiable, then (1) is not necessary to secure the validity of his argument. Premise (1*), (WC) is the correct analysis of the concept of a priori justification, is sufficient. But, if (WC) is sufficient to secure the validity of his argument, then there is a strong dialectical reason to abandon (SC). Embracing (SC) requires offering some rationale for imposing higher standards on a priori justification than those required by the background theory of justification. Why incur the burden of supporting (SC) if you can get what you want for free?

Kitcher considers an alternative account of the process that is alleged to produce a priori knowledge. Rather than focusing on the highly specialized knowledge of innovative mathematicians, he (2000, 75) turns to the more mundane arithmetical knowledge of the folk and considers the view that “We acquire the basic arithmetical concepts at teacher’s knee, and from then on carry within us the ability to apprehend such things as that $2 + 1 = 3$.” Here a proponent of (WC) appears to be in a position to maintain that given the strong track record of the process of forming arithmetical beliefs on the basis of our grasp of concepts, doubts about the reliability of the process do not arise and such knowledge is a priori.

Kitcher disputes this contention by asking us to consider the innovators who introduced the framework of concepts involved in our present mathematical knowledge. What account can we provide of their knowledge? Kitcher (2000, 76) offers three options: (1) it is grounded in intuition; (2) it is grounded in their grasp of mathematical concepts; and (3) it is not a priori: “they are warranted in proposing new concepts and principles through an often lengthy process of demonstrating that their new ideas play a fruitful role within inquiry.” Kitcher rejects options (1) and (2). In the case of option (1), he reverts back to his earlier argument that the justification conferred on beliefs by the process of intuition is undermined by the fact that the exercise of this

process has had variable results. In the case of option (2), he (2000, 76) argues that the justification conferred on beliefs by the process of exercising one's grasp of mathematical concepts is also undermined by the fact that "the history of mathematics is full of unfruitful, even incoherent, specifications of mathematical concepts." Kitcher endorses option (3). He claims, however, that if option (3) is conjoined with the claim of proponents of (WC) that the mundane mathematical knowledge of the folk is a priori, we are led to the paradoxical conclusion that the mathematical knowledge of the folk is a priori but that of the conceptual innovators is not.¹²

We argued earlier that the fact, if it is a fact, that the exercise of mathematical intuition has had variable results does not entail that one's exercise of the process ought always to be accompanied by doubts as to whether one has exercised it correctly. Therefore, Kitcher's contention that the justification conferred on mathematical beliefs by the process of intuition is always undermined is without basis. Kitcher's contention that the justification conferred on mathematical beliefs by one's grasp of mathematical concepts is undermined rests on a similar error. The fact, if it is a fact, that the process of forming beliefs based on a grasp of mathematical concepts has had variable results does not entail that one's exercise of that process ought always to be accompanied by doubts as to whether one has exercised it properly since one need not either believe or be justified in believing that the exercise of the process has had variable results. Therefore, Kitcher fails to show that the mathematical knowledge of the conceptual innovators cannot be grounded either in intuition or in their grasp of mathematical concepts.

IIb

Kitcher's (2000, 77) second contention is that (WC) fails to capture a feature of the traditional conception of a priori knowledge: "the tradition ascribes to a priori knowledge the

functional significance of being in a position to prescribe to future experience; knowledge that prescribes to future experience is irrefutable by future experience.” Let us grant that the tradition ascribes to a priori knowledge the functional significance of prescribing to future experience. This observation, taken by itself, does not provide a basis for favoring (SC) over (WC). The reason is straightforward: Kitcher has not addressed whether this feature is constitutive of the traditional concept of knowledge. If it is constitutive of the traditional concept of knowledge, then it is a feature common to both a priori and empirical knowledge. It is not a feature that differentiates a priori knowledge from empirical knowledge and, hence, it is not constitutive of the a priori.

What is the traditional concept of knowledge? It is generally accepted that the traditional concept is Cartesian foundationalism:

Historically it has been common to require of the foundations of knowledge that they exhibit certain ‘epistemic immunities’, as we might put it, immunity from error, refutation, or doubt. Thus Descartes, along with many other seventeenth- and eighteenth-century philosophers, took it that any knowledge worthy of the name would be based on cognitions the truth of which is guaranteed (infallible), that were maximally stable, immune to ever being shown to be mistaken (incorrigible), and concerning which no reasonable doubt could be raised (indubitable). (Alston 1992, 146)

If this characterization is correct, then it follows that, according to the tradition, incorrigibility is a necessary feature of knowledge and, *a fortiori*, a necessary feature of a priori knowledge. But incorrigibility is not a feature that distinguishes a priori knowledge from empirical knowledge and, hence, is not constitutive of the a priori. Therefore, the fact that the tradition ascribes to a

priori knowledge the functional significance of prescribing to future experience, taken by itself, provides no basis for preferring (SC) over (WC).

Moreover, once we clearly distinguish between the requirements of the traditional Cartesian conception of knowledge and the requirements of the a priori, we are in a position to see that (WC) is actually more consonant with the tradition than (SC). The reason is again straightforward. Given the Cartesian concept of knowledge, the distinctive condition of (SC) is both redundant and misleading. It is redundant since the Cartesian concept of knowledge guarantees that it is satisfied. It is misleading since it suggests that irrefutability by future experience is a feature that differentiates a priori knowledge from empirical knowledge. Kitcher has been led astray by his failure to distinguish between the features of the traditional Cartesian concept of knowledge and those of the a priori.

IIc

Kitcher's next line of argument is to suggest that (WC) is too weak. Consider the following thought experiment. Suppose that a cubical die, which is made of some homogeneous material and whose faces are numbered 1 through 6, is rolled once. What is the chance that the uppermost face will be the one numbered 6? One might reason as follows: The material is homogeneous. Therefore, the situation is symmetrical with respect to the six faces; nothing favors any one face over the others. One of the numbered faces will be uppermost. Therefore, the probability that it will be the one numbered 6 is $1/6$. Kitcher maintains that the process involved in this thought experiment is nonexperiential and, moreover, that it meets reliabilist standards:

In fact, given the way the world works, processes of viewing symmetry as a guide to chance regularly generate true beliefs—most dice made of uniform materials conform to this probability. So if we consider the relevant type of process to be the class of thought-experiments in which one employs considerations of symmetry to conclude that the chance that an n -sided die made of uniform materials will land on any particular face will be $1/n$, then it seems that, in the relevant alternatives, cases in which such processes are used to generate beliefs in worlds very like ours, the processes will regularly yield true beliefs. (Kitcher 2000, 78)

Therefore, according to Kitcher, (WC) has the consequence that the conclusion in question is known a priori.

Kitcher's contention that the conclusion in question meets reliabilist standards is tenuous. His characterization of the process that produces the conclusion involves two elements: (a) a method of arriving at that conclusion, thought experiments employing considerations of symmetry, and (b) the conclusion itself, the belief that the chance that an n -sided die made of uniform materials will land on any particular face is $1/n$. In support of the contention that this process meets reliabilist standards, Kitcher argues that, in the actual world and in all relevant alternatives, when that method of belief formation is employed to arrive at the conclusion that the chance that an n -sided die made of uniform materials will land on any particular face is $1/n$, the conclusion is generally true. The argument falls short of its goal. It establishes at most that the belief forming process in question is *locally* reliable.¹³ According to reliabilism, however, a belief is justified only if it is produced by a belief forming process that is *globally* reliable: one that is reliable with respect to the *full range* of beliefs that it produces. But Kitcher has not

articulated the full range of beliefs produced by the process in question, let alone shown that they are regularly true.¹⁴

But suppose that it should turn out that the belief forming process in question is globally reliable. Why is this result problematic for (WC)? Kitcher (2000, 79) makes two related claims: “The obvious danger is that this will set the Weak conception at variance with the classical view of the bounds of apriority—if it delivers this result, then it cheapens the notion.” The first claim is that (WC) delivers results that are at variance with the classical view. This claim is mistaken because it fails to distinguish between the requirements of (WC) and the background theory of knowledge in which it is embedded. If (WC) is embedded within the classical Cartesian theory of knowledge then it does not deliver results at variance with the classical theory. In order for a belief to be justified or known a priori within a Cartesian theory of knowledge, it must meet the general conditions on justification and knowledge imposed by that theory. Since Kitcher’s example does not meet those conditions—it is neither infallible nor incorrigible nor indubitable—it is not justified or known a priori on the classical view. Kitcher generates the appearance of variance between (WC) and the classical theory by embedding (WC) within a reliabilist theory of knowledge. The variance is due entirely to the difference in the background theories of knowledge in which (WC) is embedded. It is not due to (WC).

The second claim is that (WC) cheapens a priori knowledge. In order to assess this charge, it is necessary to locate more precisely the difference between (SC) and (WC). If (SC) and (WC) are both embedded in a Cartesian theory of knowledge, then (SC) is satisfiable only if (WC) is satisfiable. Within a Cartesian theory, there is no difference between (SC) and (WC) since the distinctive condition of (SC), indefeasibility by experience, is redundant. If (SC) and

(WC) are embedded in a non-Cartesian theory, such as reliabilism, then there is a difference between them. In order to locate more precisely this difference, consider some nonexperiential process Φ that produces in S some belief that p. Either Φ is globally reliable or not. If it is not, then S's belief that p is not justified on either (SC) or (WC). If it is, then either that justification is defeated or not. If it is, then S's belief that p is not justified on either (SC) or (WC). If Φ is globally reliable and S's justification is not defeated, then either S's justification is defeasible by experience or it is not. If not, then S's belief that p is justified a priori on both (SC) and (WC). If it is, then S's belief that p is justified a priori on (WC), but it is justified a posteriori on (SC). So, assuming that both conceptions are embedded in a reliabilist theory of knowledge, we get the following result, called the *Parity Principle*:

(PP) If S's belief that p is justified a priori on (WC) but not on (SC), then it is justified a posteriori on (SC).

The Parity Principle entails that a priori knowledge is as cheap on (WC) as a posteriori knowledge is on (SC). This consequence is problematic only if one holds that the standards for a priori knowledge are higher than those for a posteriori knowledge. But the question that (SC) must address is: What justifies the higher standards?

III

The second part of Kitcher's case against (WC) derives from a shift in his background theory of knowledge. He (2000, 80) now rejects reliabilism in favor of a socio-historical conception of knowledge:

On my *socio-historical* conception of knowledge, the knowledge we have today isn't simply a matter of what we have experienced or thought during the course of our lives,

but is dependent on the historical tradition in which we stand and on the social institutions that it has bequeathed to us.

The socio-historical conception represents a break from the epistemological tradition since, according to Kitcher, most epistemologists from Descartes to the present maintain that our knowledge is tradition-independent, where “a person’s knowledge is independent of socio-historical tradition just in case that person could have had the knowledge, even given socialization in a different tradition, provided only that the socialization made it possible to entertain the proposition known” (Kitcher 2000, 81).

Kitcher maintains that the socio-historical conception of knowledge bears on the dispute between (SC) and (WC) in two ways. First, it shows that (SC) underwrites the classical view that a priori knowledge is tradition-independent, but (WC) results in the tradition-dependence of such knowledge. Second, it reveals that contemporary mathematical knowledge depends on the experiences of our ancestors, which has two significant consequences: (a) it undercuts the efforts of proponents of (WC) to preserve the a priori status of mathematics; and (b) it reveals that the primary issue regarding mathematical knowledge is not whether it is a priori but whether it is tradition-independent.

IIIa

Kitcher contends that (SC) underwrites the classical view that a priori knowledge is tradition-independent, but (WC) results in the tradition-dependence of such knowledge. In support of this contention, Kitcher alleges that (a) Frege provided an a priori route to mathematical knowledge that he regarded as tradition-independent, but (b) if Frege were employing (WC) that route would be tradition-dependent:

Suppose that the conception of a priori knowledge employed in these discussions were just the Weak conception. Then there are possible lives, given which processes that would normally warrant a belief in various mathematical propositions would fail to do so. Now imagine a historical tradition whose members have such experiences in the generation that precedes ours. There are two possibilities: in socializing us they either respond to the subversive experiences by explicitly identifying certain processes as unreliable . . . or they do not. If they do, then we are not warranted in believing parts of mathematics on the basis of the process, any more than someone who has been told about mirages is warranted by his perceptions in believing that there is an oasis in the distance (even though there may be one there). If they do not, then we are still not warranted, for our epistemic situation is akin to that of people reared in a community of dedicated clairvoyants who ignore evidence that their chosen methods are unreliable. . . . Hence, . . . our knowledge turns out to be tradition-dependent. (Kitcher 2000, 82)

On the other hand, according to Kitcher, if (SC) is adopted, this argument is blocked and the tradition-independence of mathematical knowledge is preserved.

Kitcher's argument is not transparent. I offer the following reconstruction. Let us begin with the simplifying assumption that, according to Frege, a single process Φ is the source of all mathematical knowledge. Consider now some mathematical proposition that p and assume that it is justified a priori by Φ :

(A1) S's belief that p is justified a priori by Φ .

In the first sentence of the quoted passage, Kitcher invites us to assume that Frege is employing (WC) in his discussion of mathematical knowledge:

(A2) (WC)

On the basis of this assumption, he concludes in the second sentence of the quoted passage that

(C1) Therefore, the justification conferred on S's belief that p by Φ is defeasible by experience.

Let us call the type of experiences that would defeat the justification conferred on someone's belief that p by Φ were that person to have them *subversive*. Kitcher invites us to imagine that S's socializers have such subversive experiences. Since those experiences would provide evidence that Φ is unreliable, he concludes that

(C2) Therefore, it is possible that S's socializers have experiential evidence that Φ is not reliable.

He then argues by dilemma in support of the following epistemic principle:

(EP) If S's belief that p is produced by a reliable process Φ and S's socializers have evidence that Φ is not reliable, then the justification conferred on S's belief that p by Φ is defeated.¹⁵

It follows from (EP) that if S's belief that p is justified a priori by Φ and S's socializers have experiential evidence that Φ is not reliable, then

(C3) The justification conferred on S's belief that p by Φ is defeated.

Since the justification of S's belief that p depends on whether or not her socializers have evidence that Φ is not reliable, it follows that

(C4) S's justification for the belief that p is tradition-dependent.

Kitcher's claim that (WC) leads to the tradition-dependence of a priori knowledge is mistaken. The initial step of his argument rests on a critical misunderstanding of (WC). Kitcher

assumes that it is a consequence of (WC) that if S's justification for the belief that p is a priori, then S's justification is defeasible by experience. (WC), however, does not entail that a priori justification is defeasible by experience. It entails only that defeasibility by experience is compatible with a priori justification.¹⁶ Since the inference from (A1) and (A2) to (C1) is invalid, Kitcher's argument fails to establish that (WC) leads to the tradition-dependence of a priori knowledge.¹⁷

Kitcher's (2000, 82) contention that his argument is blocked if (SC) is adopted, thus ensuring the tradition-independence of a priori knowledge, is also mistaken. Let us begin by considering some mathematical proposition that p and assume that it is justified a priori by Φ :

(A1) S's belief that p is justified a priori by Φ .

Suppose that we replace (A2) in Kitcher's argument by

(A2*) (SC)

The conjunction of (A1) and (A2*) entails

not-(C1) Therefore, the justification conferred on S's belief that p by Φ is not defeasible by experience.

But not-(C1) does not entail

not-(C4) S's justification for the belief that p is tradition-independent.

Kitcher has overlooked the possibility of nonexperiential subversive evidence. If nonexperiential evidence that Φ is not reliable is possible—say, for example, that the exercise of Φ frequently seems to lead to paradoxical or inconsistent results—then it follows that

(C2*) Therefore, it is possible that S's socializers have nonexperiential evidence that Φ is not reliable.

But if S's belief that p is justified a priori by Φ and S's socializers have evidence Φ is not reliable then, given (EP), it follows that

(C3) The justification conferred on S's belief that p by Φ is defeated.

Since the justification of S's belief that p depends on whether or not her socializers have nonexperiential evidence that Φ is not reliable, it follows that

(C4) S's justification for the belief that p is tradition-dependent.

The moral is clear. In order for a priori knowledge to be tradition-independent, a priori justification must be indefeasible. This leads to a second, more fundamental, criticism of Kitcher's argument.

Kitcher's goal is to show that if Frege were employing (WC), then his account of mathematical knowledge would be tradition-dependent. But in order to derive that result, he embeds (WC) within a background theory of knowledge, socio-historicism, which he acknowledges represents a departure from the epistemological tradition that includes Frege. If we make the plausible assumption that Frege's background theory of knowledge is Cartesian foundationalism, then it follows that for Frege:

(A0) If S's belief that p is justified, then S's justification for the belief that p is indefeasible.

But if we add (A0) to Kitcher's argument, the conjunction of (A0), (A1) and (WC) entails not-(C1), and the argument stops at that point. Hence, (A0) is sufficient to preserve the tradition-independence of a priori knowledge. Moreover, as we saw in the previous paragraph, (A0) is also necessary to preserve the tradition-independence of a priori knowledge. So Kitcher is faced with a dilemma. Either Frege's background theory of knowledge entails (A0) or it does not. If it

does not, then neither (SC) nor (WC) preserves the tradition-independence of a priori knowledge. If it does, then both (SC) and (WC) preserve the tradition-independence of a priori knowledge.

We can now generalize and sharpen the basic criticism of (SC) introduced in section I. Either (SC) is embedded in the classical (or traditional) theory of knowledge or it is not. The classical theory of knowledge, Cartesian foundationalism, entails (A0). Within a theory of knowledge that entails (A0), condition (b) is both unnecessary and misleading. It is unnecessary, because the possibility of experiential defeaters for beliefs justified a priori is ruled out by the general theory of knowledge. It is misleading because it implies that indefeasibility by experience is a feature that differentiates a priori justification from empirical justification; but, according to the classical theory of knowledge, that feature is common to both. Condition (b) is necessary only if (SC) is embedded in a nonclassical theory of knowledge—i.e., one that does not entail (A0). But if a general theory of knowledge does not entail (A0) and condition (b) distinguishes a priori justification from empirical justification, then it follows that empirical justification is compatible with defeasibility by experience. Therefore, within a nonclassical theory of knowledge, condition (b) imposes a higher standard on a priori justification than the general theory of knowledge imposes on empirical justification. In the absence of some compelling supporting argument, the higher standard is *ad hoc*. Therefore, (SC) is either unnecessary or *ad hoc*.

IIIb

Kitcher (2000, 84) maintains that the socio-historical conception of knowledge reveals that contemporary mathematical knowledge depends on the experiences of our ancestors in two ways: (1) “the ultimate starting points lie in those scattered perceptions that began the whole

show,” and (2) “the warranting power of the contemporary processes depends on the division of labour and the long sequence of experiences that have warranted our ancestors, and now us, in making that division.” This revelation has two significant consequences. First, philosophers who were “concerned to save the apriority of mathematical knowledge by weakening the conception so that it only demands (a), have failed to see that the warranting power of the processes of thought they take to underlie mathematical knowledge depends on the *experiences of those who came before us in the mathematical tradition*” (Kitcher 2000, 84). Second, “we’ll recognize that the issue isn’t one of apriorism versus empiricism, but of apriorism versus historicism, and here the interesting question is whether one can find, for logic, mathematics, or anything else, some tradition-independent warrant, something that will meet the requirements that Descartes and Frege hoped to satisfy—in short, something that will answer to the Strong conception” (Kitcher 2000, 85).

Let us grant Kitcher’s socio-historical conception of knowledge and his claim that our mathematical knowledge depends on the experiences of our ancestors, and focus on the alleged consequences of these insights. His contention that proponents of (WC) who defended the a priori status of mathematical knowledge overlooked the fact that the experiences of our ancestors play a role in justifying our mathematical beliefs disregards the context of their defense. They were responding to Kitcher’s (1983) leading argument, which has the following structure:

1. Reliabilism is the correct general theory of knowledge.
2. (SC) is the correct analysis of a priori justification.
3. The justification conferred on mathematical beliefs by alleged a priori processes is defeasible by experience.

4. Therefore, mathematical knowledge is not a priori.

Kitcher's critics conceded premises (1) and (3), but argued that (2) is false. Two observations are in order here. First, showing that (2) is false is sufficient to show that the argument is unsound. Second, the critics did not address the role of the experiences of our ancestors because (1) entails that they play no role in the justification of our mathematical beliefs.

Kitcher's second contention is that the important issue regarding mathematical knowledge is apriorism versus historicism, and that the interesting question is whether there is some tradition-independent warrant for mathematics. This contention is rooted in a conceptual confusion: a failure to distinguish between the requirements of the general concept of knowledge and those of the concept of a priori knowledge. The important epistemological question regarding mathematical knowledge cannot be framed as a choice between apriorism and historicism because historicism is a thesis about the nature of knowledge in general; apriorism is not. Similarly, the traditional debate over a priori knowledge cannot be cast as a debate concerning whether there is tradition-independent warrant for mathematics since tradition-dependence is a consequence of the socio-historical conception, which is a thesis about knowledge in general, whereas the traditional debate concerned whether there is any interesting difference between mathematical knowledge and knowledge of the external world.

The central claim of socio-historical conceptions of knowledge is that the justification of a person's beliefs sometimes depends on the cognitive states and processes of that person's intellectual ancestors. Kitcher (2000, 81-82) typically contrasts the socio-historical conception with "synchronic" conceptions of justification, which hold that the justification of a person's beliefs depends only on that person's cognitive states and processes. Foundationalism and

coherentism, according to Kitcher, provide examples of such theories. Hence, the debate between socio-historical theories and synchronic theories is a debate over the general requirements for justification and knowledge.

The debate between proponents of the a priori and proponents of empiricism is a debate within the general theory of knowledge and justification. The basic question can be formulated as follows. Consider the body of beliefs that meet all the conditions on justification set by the background theory of knowledge. Can they be divided into two classes based on some interesting difference in the role that experience plays in their justification? The dispute between proponents of (SC) and (WC) is over how to characterize that difference. That dispute can be formulated within both socio-historical and synchronic theories of knowledge. Within a synchronic theory of knowledge, which holds that a person's justification derives solely from that person's cognitive states and processes, (WC) holds that S's belief that p is justified a priori iff the cognitive states and processes of S that justify the belief that p are exclusively nonexperiential. (SC) adds a further condition: S's nonexperiential justification is not defeasible by S's experiences. Within a socio-historical theory, which holds that a person's justification can extend to the cognitive states and processes of that person's intellectual ancestors, (WC) holds that S's belief that p is justified a priori iff the cognitive states and processes of S *and S's intellectual ancestors* that justify the belief that p are exclusively nonexperiential. (SC) adds the further condition: S's nonexperiential justification is not defeasible by S's experiences *or those of S's intellectual ancestors*. Hence, whichever conception of knowledge one adopts, the two traditional questions about the a priori can be posed: What is the correct analysis of the concept

of a priori knowledge? Is mathematical knowledge a priori—i.e., does it satisfy the conditions in the analysis?

Kitcher's contention that the interesting question about mathematics is whether one can find for it some tradition-independent justification that meets the requirements of Descartes and Frege cannot be right for similar reasons. The tradition-dependence of justification is a consequence of the socio-historical conception of justification. On the other hand, the tradition-independence of knowledge is a consequence of the synchronic conception of justification. Hence, the debate over whether justification is tradition-independent is a debate about the general conditions for justification; it is not a debate about the nature of mathematical justification in particular. Consequently, when Frege claims that the justification of mathematical propositions is different from the justification of propositions about the external world—that the former is a priori and the latter is not—his claim cannot be that the justification of mathematical propositions is tradition-independent but the justification of propositions about the external world is not. His commitment to Cartesian foundationalism ensures that the justification of both is tradition-independent. His claim is that although the justification of both mathematical propositions and propositions about the external world is tradition-independent, there is an important difference between the role that experience plays in the justification of each. As we argued earlier, (WC) best captures that difference.

IV

Let us step back and ask: What conclusions can we draw about the current debate over the nature of mathematical knowledge? The traditional debate centers around the question: Is mathematical knowledge a priori? Kitcher (1983) addresses this question directly: he defends the

view that mathematical knowledge is not a priori by offering an argument whose linchpin is (SC). He (2000, 85) now rejects the importance of that question because he is pessimistic about providing an analysis of the concept of a priori knowledge:

It seems to me that the discussions of the past decades have made clear how intricate and complex the classical notion of the a priori is, and that *neither* the Strong conception *nor* the Weak conception (nor anything else) can provide a coherent explication.

The first conclusion that we can draw is that Kitcher's pessimism is unwarranted.

The difficulty of providing a clear and coherent account of the classical conception of the a priori stems in large part to a failure to distinguish the conditions constitutive of the concept of a priori knowledge from those constitutive of the more general concepts of knowledge and justification. This point emerges in two different ways in Kitcher's discussion. First, the weaknesses that he attributes to (WC) are consequences of the reliabilist theory of knowledge in which he embedded his original discussion of the a priori. Second, the features of the classical conception of the a priori that Kitcher alleges are captured only by (SC) are features of the Cartesian foundationalist theory of knowledge in which it is embedded. If we carefully distinguish between the requirements of the different background theories of knowledge introduced by Kitcher's discussion and those of the a priori, we can see that (a) Kitcher has offered no cogent criticism of (WC), (b) he has not shown that (WC) is at odds with the classical conception of the a priori, and (c) he has offered no independent support for (SC). Hence, there is no basis to maintain that (WC) fails to provide an accurate and coherent analysis of the classical conception of a priori knowledge or that (SC) more accurately captures some features of that conception.

Kitcher's pessimism regarding the concept of the a priori leads him to conclude that we should move beyond the traditional debate regarding mathematical knowledge. It is not important whether mathematical knowledge is a priori:

The important point is to understand the tradition-dependence of our mathematical knowledge and the complex mix of theoretical reasoning and empirical evidence that has figured in the historical process on which current mathematical knowledge is based.

(Kitcher 2000, 85)

The second conclusion that we can draw is that Kitcher's new account provides no reason to move beyond the traditional question regarding mathematical knowledge.

Kitcher's new account of mathematical knowledge can be formulated as follows:

1. Socio-historicism is the correct general theory of knowledge.
2. The experiences of our ancestors play a role in the justification of our mathematical beliefs.
3. Therefore, our mathematical knowledge is based on a mix of theoretical reasoning and empirical evidence.

Once we recognize that the concept of a priori knowledge can be articulated within a socio-historical theory of knowledge, we can also see that Kitcher's new account of mathematical knowledge, when conjoined with (WC), offers a direct answer to the traditional question regarding such knowledge:

1. Socio-historicism is the correct general theory of knowledge.
2. (WC) is the correct analysis of a priori justification.

3. The experiences of our ancestors play a role in the justification of our mathematical beliefs.
4. Therefore, our mathematical knowledge is not a priori.

So why not endorse (WC) and claim victory?

The third conclusion that we can draw is that Kitcher's claims about the role of experience in justifying the mathematical beliefs of our ancestors raise philosophical issues that parallel those raised in the traditional debate regarding the nature of mathematical knowledge. His first claim is that the elementary mathematical knowledge of our early ancestors is justified by ordinary sense perception. This claim applies a familiar Millian account to the mathematical knowledge of our early ancestors. But the very same questions that can be (and have been) raised regarding whether Mill has shown that *our* mathematical knowledge is empirical can be raised with respect to Kitcher's claims about the mathematical knowledge of *our ancestors*. For example, suppose that Mill provides a coherent empiricist account of our ordinary mathematical knowledge and that of our ancestors. It does not follow that our mathematical knowledge or that of our ancestors is not a priori unless Mill can rule out the possibility of epistemic overdetermination—i.e., the possibility that our mathematical beliefs are justified both experientially and nonexperientially.¹⁸

Kitcher's second claim is that the institutionalization of a division of labor in the early development of modern science in which some members were given the task of developing new mathematical concepts and principles plays a role in the justification of the mathematical beliefs of our ancestors. Evaluating this claim is more challenging since he is not fully explicit about

how this epistemic division of labor shows that mathematical beliefs are justified by experience.

His (2000, 84) clearest articulation comes in the following passage:

We have learned, *from experience*, that having a group of people who think and scribble, who proceed to extend and articulate mathematical languages in the ways that mathematicians find fruitful and who provide resources for empirical science is a good thing, that creating this role promotes our inquiry.

Kitcher emphasizes here that we have learned from experience that a division of labor promotes fruitful inquiry. But, surely, from the premise that experience shows that a division of labor promotes fruitful mathematical inquiry, it does not follow that experience plays any role in the justification of mathematical beliefs. But perhaps Kitcher is here stressing the role of mathematics in promoting scientific inquiry and suggesting that this role is essential to the justification of mathematical beliefs. This reading of Kitcher introduces a familiar Quinean theme to the effect that the applications of mathematical propositions in empirical science play an essential role in their justification. Once again, the same questions that can be (and have been) raised regarding whether Quine has shown that *our* mathematical knowledge is empirical can be raised with respect to Kitcher's claims about the mathematical knowledge of *our ancestors*. For example, does the Quinean picture provide an accurate representation of our actual mathematical practices or is yet another philosophical "rational reconstruction" of a body of human knowledge of the sort that Quine and Kitcher explicitly reject?¹⁹

So, in the end, Kitcher's circuit through socio-historical theories of knowledge returns the current debate about mathematical knowledge to familiar territory. (WC) provides a coherent articulation of the concept of the a priori that is consonant with both the classical conception of

knowledge and Kitcher's socio-historical conception. His new account of the mathematical knowledge in conjunction with (WC) entails that mathematical knowledge is not a priori. The soundness of his argument rests on two familiar issues regarding mathematical knowledge. Are the experiences that are typically involved in the genesis of our mathematical beliefs (and those of our ancestors) essential to their justification?²⁰ Are the empirical applications of mathematical propositions essential to their justification?²¹

University of Nebraska-Lincoln

REFERENCES

- Alston, William. (1992). "Foundationalism." In *A Companion to Epistemology*, eds. J. Dancy and E. Sosa. Oxford: Basil Blackwell.
- Burge, Tyler. (1993). "Content Preservation." *Philosophical Review* 102: 457-488.
- Casullo, Albert. (1988). "Revisability, Reliabilism, and A Priori Knowledge." *Philosophy and Phenomenological Research* 49: 187-213.
- Casullo, Albert. (2005). "Epistemic Overdetermination and A Priori Justification." *Philosophical Perspectives* 19: 41-58.
- Casullo, Albert. (2003). *A Priori Justification*. New York: Oxford University Press.
- Eddin, Aron. (1984). "A Priori Knowledge for Fallibilists." *Philosophical Studies* 46: 189-197.
- Goldman, Alvin. (1976). "Discrimination and Perceptual Knowledge." *Journal of Philosophy* 73: 771-791.
- Goldman, Alvin. (1979). "What Is Justified Belief?" In *Justification and Knowledge*, ed. G. Pappas. Dordrecht: D. Reidel Publishing Company.
- Goldman, Alvin. (1986) *Epistemology and Cognition*. Cambridge: Harvard University Press.
- Goldman, Alvin. (1999). "A Priori Warrant and Naturalistic Epistemology." *Philosophical Perspectives* 13: 1-28.
- Kitcher, Philip. (1980). "A Priori Knowledge." *Philosophical Review* 89: 3-23.
- Kitcher, Philip. (1980). "Arithmetic for the Millian." *Philosophical Studies* 37: 215-236.
- Kitcher, Philip. (1983). *The Nature of Mathematical Knowledge*. New York: Oxford University Press.

- Kitcher, Philip. (2000). "A Priori Knowledge Revisited." In *New Essays on the A Priori*, eds. P. Boghossian and C. Peacocke. Oxford: Oxford University Press.
- Maddy, Penelope. (1997). *Naturalism in Mathematics*. Oxford: Oxford University Press.
- McGinn, Colin. "A Priori and A Posteriori Knowledge." *Proceedings of the Aristotelian Society* 76 (1975/76): 195-208.
- Putnam, Hilary. (1983) "'Two Dogmas' Revisited." In his *Realism and Reason: Philosophical Papers, Vol. 3*. Cambridge: Cambridge University Press.
- Pust, Joel. (2002). "Kitcher on Tradition-Dependent A Priori Warrant." *The Philosophical Quarterly* 52: 375-376.
- Quine, W. V. (1963). "Two Dogmas of Empiricism." In his *From a Logical Point of View, 2nd ed revised*. New York: Harper and Row.
- Summerfield, Donna. (1991). "Modest A Priori Knowledge." *Philosophy and Phenomenological Research* 51: 39-66.

NOTES

1. By general theory of knowledge or justification, I mean any theory that offers an account of the conditions necessary or sufficient for knowledge or justification.
2. This is most familiar version of the reductive approach. It is embraced by those who endorse the traditional view that justification is a necessary condition for knowledge. There are other versions. For example, some theorists, such as Burge (1993), maintain that warrant is necessary for knowledge and that justification is only one species of warrant. If such a theorist embraces the reductive approach, the primary target of analysis is the concept of a priori warrant.
3. The classic example of a nonreductive theory-neutral analysis is inspired by logical empiricism: S knows a priori that p just in case S knows that p and p is analytic. Colin McGinn (1975/76, 198) offers what is arguably a nonreductive theory-laden analysis: x knows that p a posteriori if and only if (i) x knows that p & (ii) (\exists s) (s is x's ground or

reason for believing that p & (iii) the subject-matter of s causes x to believe that p). x knows that p a priori if and only if (i) and (ii) and not-(iii). In the case of non-inferential knowledge that p, 's' is replaced by the statement that p. McGinn's analysis appears to presuppose some version of the causal theory of knowledge in the case of a posteriori knowledge. My analysis (Casullo, 2003) is an example of a reductive theory-neutral analysis: S knows a priori that p just in case S's belief that p is nonexperientially justified and the other conditions on knowledge are satisfied. Finally, Kitcher's (1983) analysis, which is presented in section 1, is an example of a reductive theory-laden analysis

4. The analysis of a priori warrant was originally presented in (Kitcher, 1980). Kitcher (1983, 88) offers the following summary of his argument:

Previous chapters have shown, systematically, that the processes which apriorists take to generate our mathematical beliefs would be unable to warrant those beliefs against the background of a suitably recalcitrant experience. If apriorists are to escape this criticism on the grounds that the analysis of apriority is too strong, then they must allow that it is not necessary for an a priori warrant to belong to a type of process members of which could warrant the belief in question given any sufficient experience. To make this concession is to abandon the fundamental idea that a priori knowledge is knowledge which is independent of experience.

5. See Casullo (1988), Edidin (1984), Goldman (1999) and Summerfield (1991).
6. The first is necessary to rule out cases like that of Henry who forms the true belief that there is a barn in front of him via his reliable process of perception while driving in fake barn country. The second is necessary to rule out cases where S believes, but does not know, some necessary truth that p, since the local reliability condition is trivially satisfied in such cases. The third is necessary to rule out cases where S's belief is produced by a globally and locally reliable process Φ , but S has evidence that Φ is not reliable.
7. Goldman (1986, 106). Goldman goes on to discuss the issue of whether the belief forming process must have a sufficiently high truth ratio in the actual world or some other world in order to capture the intuitive notion of justification.
8. Pust (2002) makes this point, but also astutely notes that Kitcher is not consistent in his use of 'warrant'. There are passages that indicate that Kitcher takes warrant together with true belief to be sufficient for knowledge. Pust, however, argues that the best overall reading of Kitcher favors taking warrant to be equivalent to justification. I suggest that the source of this inconsistency is that in his earlier papers, such as Goldman (1976), he denies that justification is necessary for knowledge. But in later works, such as Goldman (1979) and (1986), he affirms that justification is necessary for knowledge and explains that his earlier denials were intended to apply only to "Cartesian" accounts of justification, according to which "a justification is an argument, defense, or set of reasons that can be given in support of a belief" (Goldman 1979, 2). Kitcher (2000, 66) also

states: “One condition that I *don't* require of a warrant is that it involve reasons that the knower could cite in justification of the belief.” This indicates that, like Goldman, Kitcher wishes to deny that the Cartesian conception of justification is necessary for knowledge and employs the term ‘warrant’ to signal this departure.

9. See Casullo (1988). Kitcher (2000, 67-68) discusses another problem of formulation that he attributes to Boghossian and Peacocke.
10. Kitcher maintains that this quotation is from Parsons (1986), but I have not been able to locate it there.
11. There is one further option available. I (Casullo 2003, 133-140) have argued that one’s justification can be undermined by evidence present within one’s epistemic community to which one has ready access. This option, however, is not promising since reliabilism does not require that such evidence be available whenever the exercise of a process has had variable results.
12. Kitcher favors option (3), the view that new mathematical concepts and principles are justified abductively by demonstrating their fruitfulness in mathematical inquiry, but he maintains that such abductive justification is not a priori. However, he offers no defense of the latter claim. Moreover, there is good reason to question it. Very generally speaking, an abductive justification of a new mathematical concept or principle (M) involves a positive and a negative aspect. (M) is positively justified if it explains other accepted principles and leads to new principles that prove fruitful in subsequent theorizing. (M) is negatively justified if attempts to show that it leads to unacceptable conclusions fail. Let us assume that (M) explains other accepted principles and leads to new principles if we can derive the accepted principles and the new principles from (M) using accepted rules of inference. If the accepted principles, inferential rules, and judgments of fruitfulness are justified a priori, there is no basis for maintaining that the positive abductive justification of (M) is not a priori. Similarly, let us assume that (M) does not lead to unacceptable conclusions if all conclusions derived from (M) using acceptable rules of inference are acceptable. Once again, if the accepted principles and rules of inference are justified a priori, there is no basis for maintaining that the negative abductive justification is not a priori. Hence, option (3) does not generate the paradoxical result that Kitcher alleges unless he can show that empirically justified beliefs play some essential role in the abductive justification of mathematical concepts or principles. We return to this issue in section IV.
13. Actually, it does not even establish that the belief forming process is locally reliable. The local reliability condition requires that the belief in question be true in *all* relevant alternatives rather than *generally* true. Kitcher’s discussion is permeated by a conflation of the global and local reliability conditions of the background theory that he endorses.

14. Kitcher is aware that there is a problem lurking in the vicinity. He (2000, 78) envisages proponents of (WC) objecting that he has mischaracterized the relevant process: “Maybe they propose that we have to consider symmetry arguments in general and to focus on worlds in which such arguments usually fail.” In response, he (2000, 78-79) argues

But even if we concede that something like this can be achieved, perhaps because our accumulated wisdom about the use of symmetry arguments casts doubt on their reliability (a move that depends crucially on treating as relevant symmetry arguments that don’t take the form of simple reflections on uniform dice), the Weak conception has to recognize that this feature of our tradition is thoroughly contingent.

In support of this contention, Kitcher invites us to imagine a world which is like ours with respect to the “the course of nature,” but in which the initial phenomena for which symmetry arguments are developed are ones for which the arguments work well and, as a result, investigators are trained to trust the results of such arguments. According to Kitcher, the investigators in the imagined world know a priori the conclusions of such arguments.

This response reveals a misunderstanding of the problem. Kitcher seems to think that the global reliability condition must be satisfied only if there is evidence that the process in question is unreliable. Given this understanding of the requirement, he can plausibly argue that the investigators in his imagined world, who have no such evidence, are justified a priori although investigators in the actual world are not. Global reliability, however, is a general condition on justification. If a belief forming process is not globally reliable, then the beliefs produced by it are not justified, irrespective of whether the cognizers employing the process have evidence that it is not reliable. Consequently, if thought experiments involving symmetry considerations are not globally reliable, then the beliefs produced by this process are not justified either for us or for the investigators in Kitcher’s imagined world.

15. Pust (2002) rejects this principle on the grounds that justification is not defeasible by evidence that one does not possess. Although I agree with Pust that (EP) is not plausible, I disagree that with his general claim that justification is not defeasible by evidence that one does not possess. For a discussion of this issue, see Casullo (2003, 133-140).
16. Consider, for example, Quine’s famous dictum that no statement is immune to revision in light of recalcitrant experience. A proponent of (WC) who maintains that mathematical knowledge is a priori is not thereby endorsing Quine’s dictum with respect to mathematical statements. Instead, the proponent maintains that the truth of the dictum is irrelevant to the question of whether mathematical knowledge is a priori; for the possibility of such experiential defeating evidence is compatible with mathematical apriorism.

17. There is an alternative line of argument available to Kitcher. Assume that

(A1*) S's belief that p is justified by some nonexperiential process Φ and the justification that Φ confers on S's belief that p is defeasible by experience.

The conjunction of (A1*) and (WC) entails

(C1*) Therefore, S's belief that p is justified a priori by Φ and the justification conferred on S's belief that p by Φ is defeasible by experience.

But if the argument that Kitcher offers from (C1) to (C4) is sound, then the argument from (C1*) to (C4) is also sound. This line of argument shows that (WC) leaves open (rather than entails) that a priori knowledge is tradition-dependent. But, as I argue in the subsequent paragraph of the paper, (SC) also leaves open the possibility that a priori knowledge is tradition dependent.

18. For a discussion of this issue, see Casullo (2005).
19. Maddy (1997, 184) maintains that the Quinean picture of mathematics is incompatible with his epistemological naturalism:

What I propose here is a mathematical naturalism that extends the same respect to mathematical practice that the Quinean naturalist extends to scientific practice. It is, after all, those methods—the actual methods of mathematics—not the Quinean replacements, that have led to the remarkable successes of modern mathematics. . . . Where Quine takes science to be independent of first philosophy, my naturalist takes mathematics to be independent of both first philosophy and natural science (including the naturalized philosophy that is continuous with science)—in short, from any external standard.

For a more general discussion of epistemological naturalism and its bearing on issues regarding the a priori status of mathematical knowledge, see Casullo (2003, 125-146).

20. Kitcher (1980) offers a defense of Mill's account. Casullo (2003, 121-123) argues that his defense fails.
21. Acknowledgments.